AN
INTRODUCTORY COURSE
IN
OPHTHALMIC OPTICS

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IN
OPHTHALMIC OPTICS

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PREFACE

The main purpose in the preparation of this volume has been to convey a working knowledge of ophthalmic optics to students and practitioners. A number of excellent books have been written on ophthalmic optics, but the presentation, as a rule, is of such a character that it does not appeal to the average student of ophthalmology. I have tried to construct a system that can be followed by one having only an elementary knowledge of mathematics; but a system which is scientifically correct, and not inconsistent with thoroughness. I hope I have succeeded.

This book is the outgrowth of my notes used in the combined lecture and laboratory course in the Graduate School of Medicine of the University of Pennsylvania.

The works of Clay, Glazebrook, Haughton, Ganot, Carhart, Prentice, Southall, Heath, Sheard, Young, Helmholtz, Donders, Landolt, von Rohr, Tscherning, Gullstrand, Howe, Johnson, Zentmayer, Laurance, Jackson, Duane, Henker and others have been freely used, either with or without variation. Care has been taken to include only that part of the subject which concerns ophthalmology.

The solutions are made as simple as possible and nearly always supplemented by experiments or ex-

amples. Some of the diagrams are original, others have been taken from various sources, but more or less modified. Uniform notations are used throughout.

I wish to take this opportunity of expressing my sincere appreciation to Drs. G. E. de Schweinitz, T. B. Holloway and F. H. Adler for their interest, encouragement and help.

A. C.

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CHAPTER I.

INTRODUCTION.

Media. Substances such as air, water and glass, which are permeable to light and through which objects can be distinctly seen, are spoken of as *optical media* and are said to be *transparent*. A medium such as ground glass or thin paper, through which light is partially or irregularly transmitted, so that distinct vision cannot be obtained, is called *translucent*. A body through which light cannot pass is *opaque*. When light enters an opaque medium it is said to be *absorbed* by it.

No medium allows all light-rays to pass through, nor does any medium totally absorb all the light. As the thickness of a transparent body increases, the percentage of light that can pass through decreases. A thin sheet of water is transparent, but very little light reaches the bottom of the sea. On the other hand, a film of metal can be made transparent by sufficiently reducing its thickness. The terms transparent and opaque, therefore, are only relative.

Media having identical optical properties at all points are called *homogeneous* or *isotropic;* those having different properties at different points are called *heterogeneous*.

Propagation of Light. By the wave theory, light is said to be propagated through the luminiferous

ether from point to point by undulations or waves. In a homogeneous transparent medium these *wave-surfaces* or *wave-fronts* are true spheres and the normals to these surfaces are straight lines. The straight lines are called *rays* and a collection of rays is called a *pencil.* In a homogeneous transparent medium light travels in straight lines.

EXPERIMENT. *Pinhole Camera.* Make a hole about one millimeter in diameter in a large sheet of cardboard or other opaque material. Place an illuminated object in front of the sheet so that the light falls directly on the aperture. If a ground-glass screen be placed behind the aperture there will be formed on the screen an inverted image of the luminous object. Vary the distance between the screen and the hole and note the increase or decrease in the size of the image. The image becomes larger in proportion as the distance of the screen from the hole is increased and the distance of the object from the hole is diminished. The shape of the opening, if small enough, does not affect the shape of the image. Two openings will form two images, but if they are close together the images will overlap and be blurred. If the opening is large only a diffused area of illumination will be seen, because of the overlapping and blurring of the many images. This is the principle of the *camera obscura* or *pinhole camera* invented by Porta. The only explanation of the phenomenon is that light travels in straight lines.

One line of direction, representing a ray or a very narrow pencil of light from a point A (Figure 1), passes through the opening and forms an image of this point at A′. Another ray passes from B to form an image at B′. Similar lines may be drawn

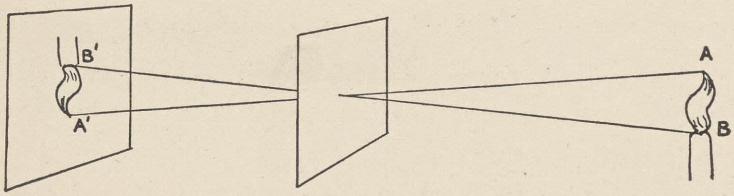

Figure 1.

from every point of the object to form an inverted image on the screen.

Shadows. If an opaque body is placed between a source of light and a screen, the light cannot

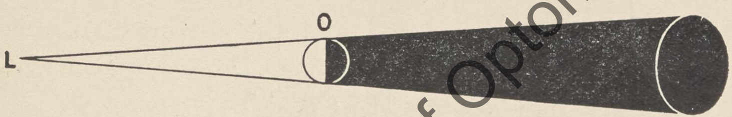

Figure 2.

penetrate into the space immediately behind the body and a shadow will be cast on the screen. This shadow is geometrically similar to the opaque body—another proof of the rectilinear propagation of light.

If the source of illumination is a point (L, Figure 2) and a spherical body, O, be placed between it and a screen, by passing a straight line through L

and tangentially around O, a circle will be traced on the screen within which no light will fall. This space is called the *umbra.*

If the source of light were a sphere instead of a point there would be a true shadow or umbra in

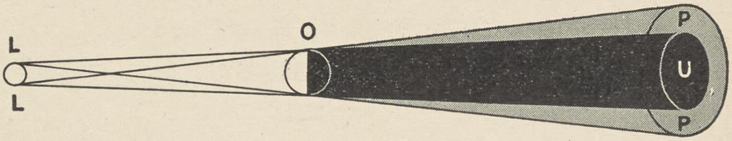

Figure 3.

the space U (Figure 3), surrounded by an area of partial shadow, P, which receives some light from parts of the luminous object L. If an eye were placed in this region only a part of the object could be seen. This partial shadow is called the *penumbra.*

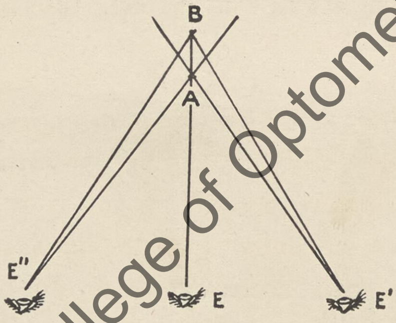

Figure 4.

Parallax. When two objects are placed one behind the other the farthest object will appear to move, relatively to the other, in the same direction as that taken by the observer.

If A and B (Figure 4) be two objects, and an observer placed at E, the two objects will appear as one because they are superimposed. When the observer moves to the right B appears to move in the same direction, and A will be to the left of B. If the eye is placed at E″ the relation changes, so that A appears to the right of B. The parallactic displacement becomes greater as the distance between the two objects increases; when there is no displacement the two objects must lie in the same plane. The phenomenon of parallax is still another proof of the rectilinear propagation of light.

The method of parallax is very often the only means of locating the position of an object in relation to that of another, hence it is very important that it be thoroughly understood.

Light Pencils. Let L (Figure 5) be a point source of light in an isotropic transparent medium. The light travels in concentric spherical waves, whose directions are indicated by the normals to the wave-surfaces. These radii of curvature, called rays, travel in all directions from L. In reality there are no rays: they merely serve as lines of direction of the light waves. A collection or pencil of rays, DLD, usually takes the form of a *cone*. The central ray, AL, LB, is called the *axis* or *chief ray* of the pencil. When the rays travel from the vertex of the cone the pencil is said to be *divergent*, and when they travel towards the vertex it is said to be *convergent*. The point where the rays meet, L, is called

a *focus*. The positive (+) sign will be used hereafter for convergent and the negative (—) sign for divergent light pencils which have been reflected or refracted. The opposite signs will be used for incident pencils.

In the diagram (Figure 5) it will be seen that the inner circle, representing the wave having travelled the least distance from L and having the small-

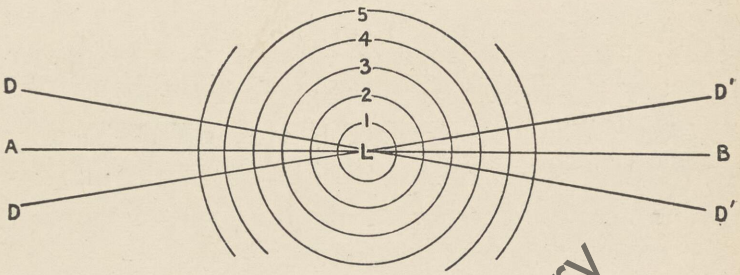

Figure 5.

est radius, has the greatest strength of curvature. As the wave travels farther from L its wave-front loses in curvature and therefore in strength until, at a distance of six meters or more, it may be considered plane; the rays, since they are perpendicular to the wave-front, are then considered parallel. A pencil of parallel rays is sometimes called a *beam*. Rays of light from a distant object, although practically parallel, are actually divergent and form a cone.

The power of the wave is the reciprocal of the distance from the focus, L, and if this distance is

measured in meters the power may be expressed in diopters.

$$\frac{1}{d} = D$$

where D represents diopters, and d the distance in meters of the wave from the focus.

For example, if the distance is 1 meter, the power is expressed by $1/1$ meter or 1 D; if the distance is 4 meters, the power is $1/4$ meter or $1/4$ D, which is usually expressed in decimals and written 0.25 D. If the distance is half a meter, the power equals $1/.5$ meter or 2 D; if a tenth of a meter, it is $1/.10$ meter or 10 D; increasing until, at the focus, the power is infinitely great.

CHAPTER II.

REFLECTION AT PLANE SURFACES.

Laws of Reflection. When a ray of light strikes a smoothly polished surface it will be regularly reflected according to certain definite laws.

Let LL′ (Figure 6) represent an incident ray of light striking a polished surface, SS, at a point O.

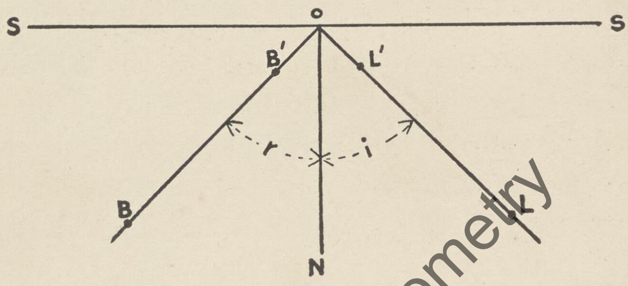

Figure 6.

The normal to the surface at this point forms an angle with LL′ which is called the *angle of incidence.* The ray LL′ is then reflected along BB′ in such a direction that the angle NOB, called *the angle of reflection, is equal to the angle of incidence.* This is the first law of reflection. The second law is that *the incident ray, the normal to the surface and the reflected ray all lie in the same plane.*

Experiment *to prove that the Angle of Incidence is Equal to the Angle of Reflection.* Draw a line, SS,

across a large sheet of paper.* Draw LL′ in an oblique direction, to meet SS at O. Erect ON, perpendicular to SS. Support a plane mirror, about 3 x 10 centimeters in size, in a vertical position, so that its silvered surface coincides with SS. Stick a pin vertically at L′, near the mirror, and another at L, as far away as the paper will allow. Now look into the mirror along the direction of BB′ and stick a pin at B′ so that it is exactly in line with the images of LL′. Place another at B in line with B′ and the images of LL′. If the eye is moved so as to look in the direction LL′ it will be seen that these pins are in line with the images of BB′. It is an important principle in optics that if a ray of light be turned backward it will travel along the same path.

Remove the mirror, join BB′ and produce this line so that, if the experiment has been carefully done, it will meet L′L on the surface at O. Prove, by measurement with a protractor, that the angles of incidence and reflection are equal.

Images. If, after reflection or refraction, rays of light from a point meet again, or appear to meet again in another point, the second point is an *image* of the first point. The first point is called the *object-point* and the rays from it are called *incident* or *object rays.* The second point is called the *image-point* and the rays, after reflection or refraction, are called the *emergent* or *image rays.*

* It will be found best to use cross-ruled or graph paper for all drawings.

An image is *real* when the emergent rays converge to, and actually meet in, the image-point. A real image can be received on a screen. If the emergent or image rays diverge, and therefore cannot meet, an *apparent* image-point will be formed where the rays would intersect if prolonged backwards in the direction from whence they came. This is

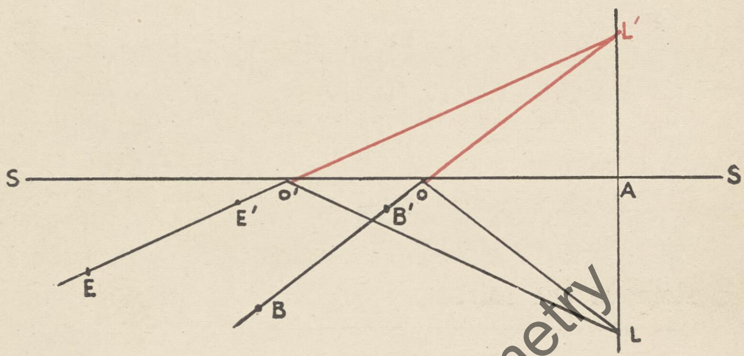

Figure 7.

called a *virtual image* because the rays do not actually, but only appear to diverge from it. A virtual image has no real existence and cannot be received on a screen.

Experiment *to find the Image of a Point in a Plane Mirror.* Draw SS (Figure 7) across the middle of a sheet of paper and place a plane mirror on this line, as was done in the last experiment. Stick a pin at L, somewhere in front of the mirror.

Looking in the mirror in the direction of B′B stick a pin at B′ and another at B so that they are in line with the image of L as seen in the mirror. Place B as far away as the paper will allow. After doing the same along E′E, remove the mirror, draw lines joining B′B and E′E, produce them until they meet behind SS in L′, then connect L′ with L. Measure the angle LAS and the distances LA and AL′.

Rays of light diverging from the object-point, L, strike the surface SS at O and O′. They are reflected in the direction B′B and E′E, still divergent, so that an eye, placed in the path of these reflected rays, receives the impression as though they came from a point L′ where they would meet if prolonged backwards. The image of a point formed by a plane mirror is virtual, and it is formed at a distance behind the mirror equal to that of the object-point in front of it and on the perpendicular to the mirror let fall at this point.

Experiment *to locate the Image of a Pin in a Plane Mirror by Parallax.* Stick a pin somewhere in front of a plane mirror. Using another pin, large enough to be seen over the top of the mirror, find the place where there is no parallactic displacement between the image seen in the mirror and the pin being used as the image-finder. Practice this until it can be done accurately.

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Experiment *to trace the Image of an Object by locating the Images of those Points which are sufficient to determine its Form.* In front of a line (SS, Figure 8) draw a triangle ABC. Erect a plane mirror as was done in the previous experiments. Insert a pin at A and locate its image behind the mirror with pins at DE and FG. Locate

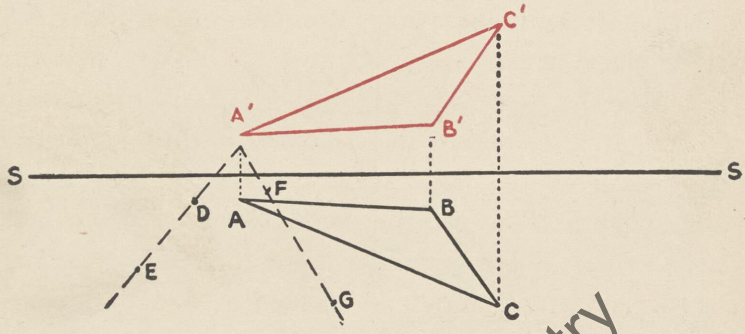


Figure 8.

the images of pins at B and C in the same way. Remove the mirror and join A'B'C', the images of A, B and C. Measure the size of the object and its image. It will be seen that the image is of the same size as the object, erect and laterally inverted. The image of an object is a collection of the images of its points.

Construction *to trace the Path of the Rays to the Eye.* Suppose LA (Figure 9) to be an object placed in front of a plane mirror, SS. To find the

image of L it is only necessary to drop a perpendicular to SS, and on this mark a point, L′, exactly as far behind the mirror as L is in front of it. Find A′ the same way. The image occupies a definite position, but the path of the rays depends on the position of the eye which receives them. This position may be anywhere in front of the mirror. The eye actually

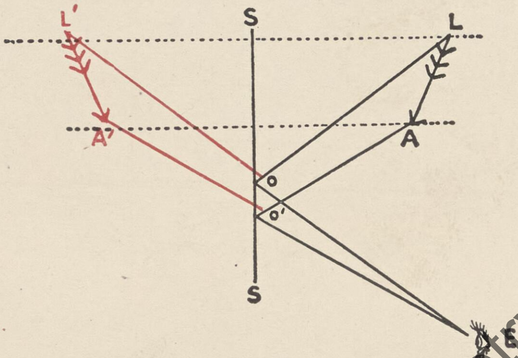


Figure 9.

sees only the reflected rays, and, since the image is projected along these rays, it is merely necessary, in order to find the path of the rays, to draw a line from the mirror to the eye in such a direction that it appears to come from the image (L′E or A′E). Since O and O′, on the reflecting surface, are the points from which the rays travel to the eye after being reflected, the incident light from L and A must have been directed towards them. This applies to each luminous point of the object between L and A.

CONSTRUCTION *to prove that an Object and all its Images, formed by two Plane Mirrors inclined at an Angle, lie in a Circle.* Let M and S (Figure 10) represent two mirrors, and let L be a luminous point. An image will be formed at L^{I} by S. L^{I}, being in front of M and because it is now an object-point, will form an image by the surface M at L^{II}.

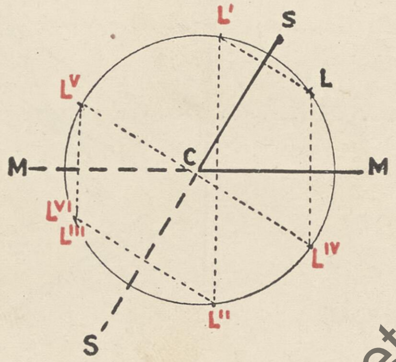


Figure 10.

L^{II}, in front of S, now forms an image at L^{III} which, since it is behind both mirrors, can form no image. Starting at L again, an image is formed by M at L^{IV}, then at L^{V} by S and finally at L^{VI} by M, behind both mirrors.

With C as center and the radius CL, describe a circle. If care has been taken, the object and all the images will be found to lie in the circle.

The number of images increases with the inclination of the mirrors. When the mirrors are at

right angles to each other three images are formed; when they are at an angle of 60 degrees, five images are formed; at 45 degrees, seven are formed. The number of images increases until, when the mirrors are parallel, it is infinite.

Diffuse Reflection. When light falls on an unpolished surface it is so reflected that it is scattered in all directions. In this way objects which are not self-luminous are made visible by the reflected light from some other source.

Absorption. When light strikes an object some is reflected, a certain amount passes through and a part is absorbed; the amount of each depending on the optical properties of the substance. Certain transparent bodies absorb light of one or more portions of the spectrum and transmit the others. For instance, yellow glass allows only yellow light to pass through, the other colors being absorbed. Opaque substances also absorb and reflect different colors—all colored objects appear to be colored for this reason. Although illuminated by white light, an object appears to be of a certain color because it diffusely reflects this color only, the others being absorbed.

CHAPTER III.

REFRACTION AT PLANE SURFACES.

When light, travelling obliquely, passes from one transparent medium into another of different density it is bent or deflected at the bounding surface, and

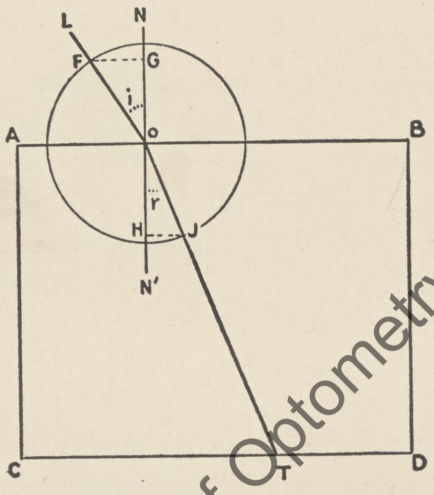


Figure 11.

is said to be *refracted*. If the incident ray is perpendicular to the surface it continues in a straight line.

EXPERIMENT. Place a piece of thick plate-glass, of about the same shape as shown in Figure 11, on a sheet of paper and draw the line ABCD around it, as close to the four sides as possible. Stick a pin at O,

against the glass, and another at L in an oblique direction. Looking along TO, through the polished edge of the slab, place T in line with the images of the pins at O and L. Remove the glass, join LOT and erect the perpendicular to AB at O. With O as center describe a circle of any radius and draw FG, the sine of the angle LON, and HJ, the sine of the angle N′OT. Measure the lengths of FG and HJ and find their ratio.

The *incident ray* is LO, and the angle LON, formed by LO and the perpendicular ON, is the *angle of incidence,* i. The refracted ray is OT, and it forms with ON′ the *angle of refraction,* r. The plane LON is the *plane of incidence,* and N′OT is the *plane of refraction.*

The velocity of light depends on the density of the medium. It is greatest in a vacuum, and its speed decreases as the density of the optical medium becomes greater. A dense medium, in which light is propagated with a low rapidity, is said to be highly refractive.

Laws of Refraction. 1. *The plane of incidence coincides with the plane of refraction.* 2. *The angles of incidence and refraction, for any two media, are so related to each other that the ratio of their sines is constant.*

Repeat the last experiment with the pin L in a different position and note that the ratio remains the same. This constant ratio is called the *index of refraction* which, being designated by n, is written:

$$n = \frac{\sin i}{\sin r}$$

The refractive index of any transparent substance is inversely proportional to the speed of light in that substance, so that if v represents the velocity in the first medium and v′ the velocity in the second medium,

$$n = \frac{\sin i}{\sin r} = \frac{v}{v'}$$

From which it may be seen that the relative index of refraction for any substance depends on the difference in the velocity of light before and after refraction by the substance.

EXPERIMENT. *Refraction through a Plate of Glass with Parallel Sides.* Repeat the last experiment; but before removing the slab insert another pin at U, as shown in Figure 12, to line with the pin at T and the images of pins O and L as seen through the edge of the glass. Draw the normals at O and T. Find the ratio of the sines of i and r and compare it with that of i′ and r′.

When the incident ray (LO, Figure 12) passes from a rare medium, such as air, into a denser medium it is bent along OT *toward* the normal, and for glass, in round numbers,

$$n = \frac{\sin i}{\sin r} = \frac{3}{2}$$

a number greater than unity, as is always the case when the light passes from a rare into a dense medium.

Now the ray OT within the glass, on striking

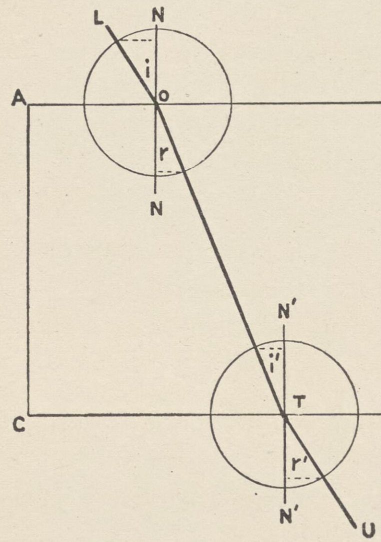


Figure 12.

the surface CD, passes into the air along TU, being bent *away* from the normal, so that for glass into air,

$$n = \frac{\sin i'}{\sin r'} = \frac{2}{3}$$

The index of refraction of glass to air is the reciprocal of the index of refraction of air to glass, so that

$$\frac{\sin i'}{\sin r'} = \frac{\sin r}{\sin i}$$

and, since NN and N′N′ are parallel, the incident and the emergent rays are parallel.

When the density of any optical medium is compared with vacuum its index of refraction is given as *absolute*. As the difference between the index of refraction for air and vacuum is very slight the refractive index for air is considered the same value as that for vacuum—unity—and

$$\sin i : \sin r :: n : 1$$

or

$$n \times \sin r = 1 \times \sin i$$

Compared with air and considering white light, the index of refraction for water is 1.33; cornea, aqueous and vitreous, 1.336; glass, 1.5 plus.

Refraction of a Pencil of Light through a Plane Surface. Repeat the experiment described on page 26 and designate the object-pin by L which we call a point source of light. Drop a perpendicular at this point to AB through S, as seen in Figure 13, and produce OT to L′ on this line.

The angles NOL and N′OT are the angles of incidence and refraction and therefore their sines are in a constant ratio. Let this ratio or refractive index be called n. In the triangle LOL′

$$LO : L'O :: \sin OL'L : \sin OLL'$$

but $OL'L = TON' = r$, the angle of refraction, because ON′ is parallel to L′S; and OLL′ is the supple-

ment to OLS which is equal to NOL, the angle of incidence, i. Therefore

$$LO : L'O :: \sin r : \sin i$$

or

$$LO : L'O :: 1 : n$$

and solving for L′O we find

$$L'O = n \times LO$$

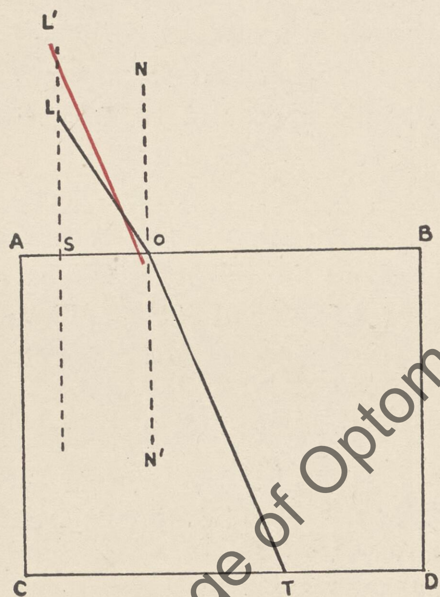


Figure 13.

The position of the point L′, therefore, depends on LO. In this, as in all that follows, we shall consider the angles very small, so that the rays lie very close to S, and writing S for O in the equation, we find for the determination of the *conjugate focus*, L′

$$L'S = n \times LS$$

If we call LS the *first conjugate focal distance* and designate it by f, and L′S the *second conjugate focal distance* and designate it by f′, we have

$$f' = n \times f$$

This means that if an eye were placed at T, in water with an index of refraction of 1.33, and an object placed at L, at a distance equal to f, the image of L would be projected to L′, along TOL′. The distance, f′, would appear to be 1.33 times greater than f.

If the focus, L, of the incident pencil were situated inside the transparent body its conjugate focus should be found in the following manner:

The angles LON′ and TON (Figure 14) are now the angles of incidence and refraction, while in the former case they were the angles of refraction and incidence, and the ratio of their sines is 1 : n instead of n : 1. As before, L being the first conjugate focus, L′ the second conjugate focus and LOT the path of the ray,

$$LO : L'O :: \sin TON : \sin LON'$$

or

$$LO : L'O :: n : 1$$

and

$$LS : L'S :: n : 1$$

or

$$L'S = \frac{1}{n} LS$$

therefore

$$f' = \frac{1}{n} f$$

From which it can be seen that if an eye were placed at T and the object at L, in water, for which

$$n = 1.33 = \frac{4}{3}$$

and

$$f' = \frac{3}{4} f$$

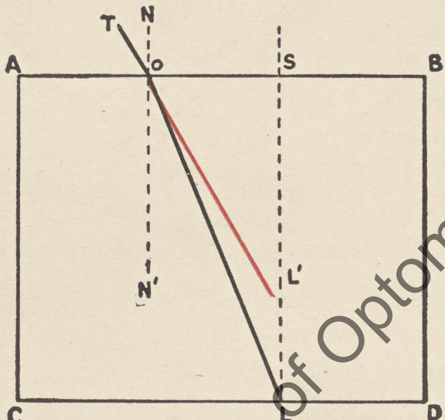


Figure 14.

the image would appear to be ¼ nearer to the surface than it actually is.

Let SS (Figure 15) be the plane surface of a body of water and LA an object placed under the surface. From L and A let fall two perpendiculars on the surface and measure LL′ and AA′ on the perpendiculars equal to ¼ their lengths. The image of LA will be seen at L′A′ by an eye placed as in the Figure.

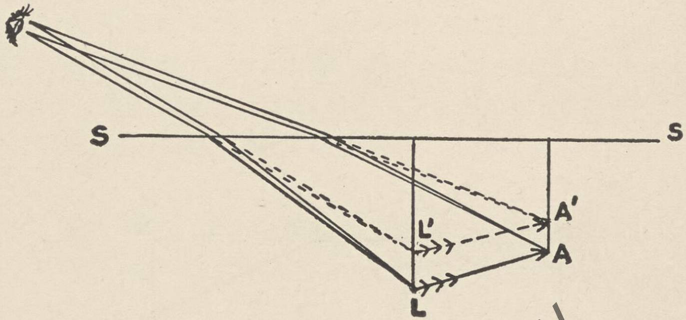


Figure 15.

Prisms. A prism, in optics, is a solid angle of glass or other transparent medium, bounded by two planes. It is a medium having two plane surfaces inclined towards each other. A plane at right angles to these two surfaces is the *principal plane;* the angle between the two faces is the *angle* of the prism; the thin edge is the *apex* and the thick end is the *base.* A ray of light passing through a prism is always deviated or bent toward the base.

EXPERIMENT. *Path of a Ray of Light through a Prism.* Place a prism or wedge of thick plate-glass on a sheet of paper and draw ABC (Figure

16) close to its sides. Stick pins close to the glass, at O and O′ so that OBO′ is isosceles. Looking along LO place L in line with the pin at O and the image of the foot of O′ seen through the prism. Now looking through the other side, in the same way as before, place T as far away as possible, to

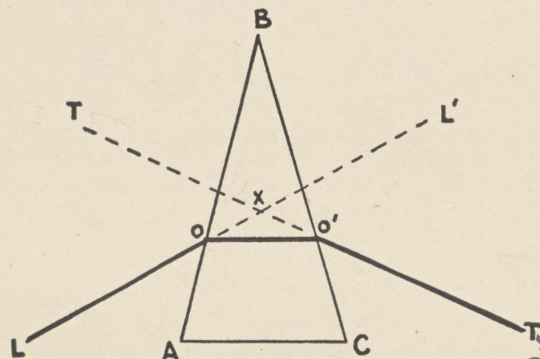


Figure 16.

line with the pin at O′ and the image of the pin at O. Remove the prism and join LOO′T. Produce LO to L′, and TO′ to T′, so that they cross at x. The incident ray, LO, is refracted at the first surface, AB, in the direction OO′, where it is again refracted at the second surface, BC, and finally leaves the prism along O′T. If LL′ is the direction of the incident ray and it emerges along TT′, the ray is deviated through the angle L x T′ which is called the *angle of deviation.* To an eye placed at T, looking through this prism, the image of L would be dis-

placed so that it would appear somewhere along TT′, directed toward the apex.

Let ABC be the principal plane of a prism, that is, a plane perpendicular to the edge and to the planes of the faces of the prism, and let LOO′T be the path of a ray of light through the prism. Draw yON and yO′N′, perpendicular to AB and BC at

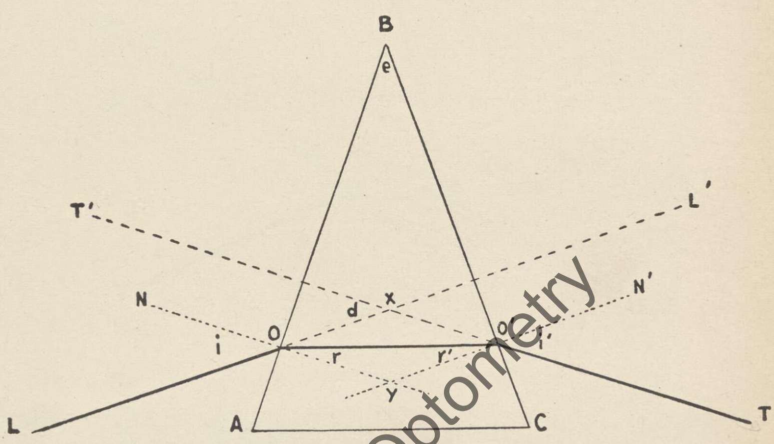


Figure 17.

the points O and O′, as is shown in Figure 17. Let the angles of incidence and refraction at the first surface, LON and yOO′, be called i and r; the corresponding angles at the second surface, TO′N′ and yO′O, be called i′ and r′; let the angle of the prism be called e; and the angle L x T′ be called d. The angle of deviation, d, measures the amount of deflec-

tion of the transmitted ray from the incident line LO.

Because Oy and O′y are perpendicular to BO and BO′, and because the quadrilateral BOyO′ is inscribable in a circle, the external angle at y is equal to OBO′ = e; but in the triangle OO′y the external angle at y is equal to the sum of the two internal and opposite angles, which are the angles of refraction r, r′; therefore

$$e = r + r' \quad . \quad . \quad . \quad . \ (1)$$

In the triangle OO′x, the external angle at x is equal to the sum of the angles xOO′ and xO′O; but it is evident from the figure that xOO′ = i — r and xO′O = i′ — r′, and the external angle at x is d; therefore

$$d = i - r + i' - r' \quad . \quad . \quad . \ (2)$$

If, now, the angle of the prism, e, be very small and the incident ray LO be nearly perpendicular to the surface, the angles i, i′, r, r′ are all small and may be considered equal to their sines; but the sines of i and i′ are to the sines of r and r′, by the law of refraction, in the ratio of n : 1; hence equation (2) becomes

$$d = (n - 1)\ 2$$

which by equation (1) becomes

$$d = (n - 1)\ e \quad . \quad . \quad . \quad (3)$$

i.e., the deviation of a ray incident nearly perpendicularly upon a prism of small angle is equal to the number n — 1 multiplied by the angle of the prism.

It is also evident from the figure that the ray LOO′T is bent away from the angle of the prism. Minimum deviation takes place when the angles of incidence and emergence are equal.

Numeration of Prisms. Prisms were first measured according to the degree of the angle. Jackson's method consists in measuring the degree of the refracting angle or according to the deviating power in the position of minimum deviation. Dennett proposed that the unit be called a centrad—a prism having the power to deviate a ray $\frac{1}{100}$ of a radian is numbered one centrad.

The standard in the United States since 1894 is the *prism diopter* which is designated by Δ. This is the method of Prentice. A prism having the power of one prism diopter will deflect a ray exactly one centimeter at a plane one meter distant, that is, the hundredth part of the radius measured on the tangent.

The property which prisms possess of separating colors is called *dispersion.* If an object be viewed through a prism of considerable power, it will be tinged with red toward the apex and with violet toward the base of the prism. Dispersion is a most important property; but the chief use of prisms in ophthalmology is for the purpose of changing the

apparent position of objects, and in these weaker prisms dispersion is not noticeable.

The following table, by Jackson, shows that the terms *centrad, prism, diopter,* and *angle in degrees* are almost interchangeable in the weaker prisms:

Deviation in Centrads	Deviation in Prism Diopters	Refracting Angle in Degrees
1	1.	1.06
2	2.	2.12
3	3.	3.18
4	4.	4.23
5	5.	5.28
6	6.01	6.32
7	7.01	7.35
8	8.02	8.38

Critical Angle. Total Reflection. When a ray of light passes from a dense into a rarer medium it is bent away from the normal, and the angle of incidence is less than the angle of refraction. When the angle of refraction is a right angle, *i.e.,* the emergent ray is parallel to the bounding surface, the angle of incidence is called the *critical angle.* If the incident ray and the normal form an angle greater than the critical angle the light cannot emerge. This is called *total reflection,* because the light is entirely reflected internally.

EXPERIMENT *to trace the Path of a Ray which has been Internally Reflected in a Prism.* Draw ABC around a prism of plate glass (Figure 18). Stick a pin at O and another at O′. Looking in the direction TO′, an image of O will be seen which has been formed by reflection at the surface AB. Insert the pin T, as far from O′ as possible, to appear in

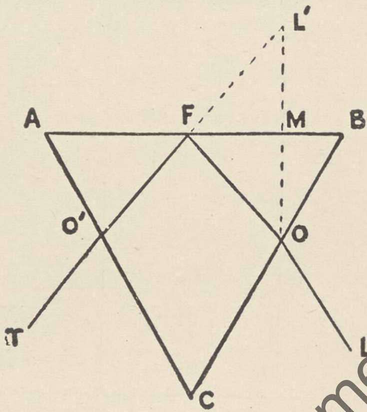


Figure 18.

line with O′ and the image of O. In the same way place the pin L by looking along the line LO (this can also be done by looking along TO′). Remove the plate, join TO′, produce it to meet AC and draw LO to meet CB. O may be considered an object-point which has been regularly reflected at the plane surface AB, and by following the method described on page 22, for the image of a point in a plane mirror, the light will be found to have taken the direction OFO′. The image will be formed at L′.

Total reflection may be demonstrated by the apparatus shown in Figure 19. On a heavy piece of cardboard or thin wood is drawn a circle with

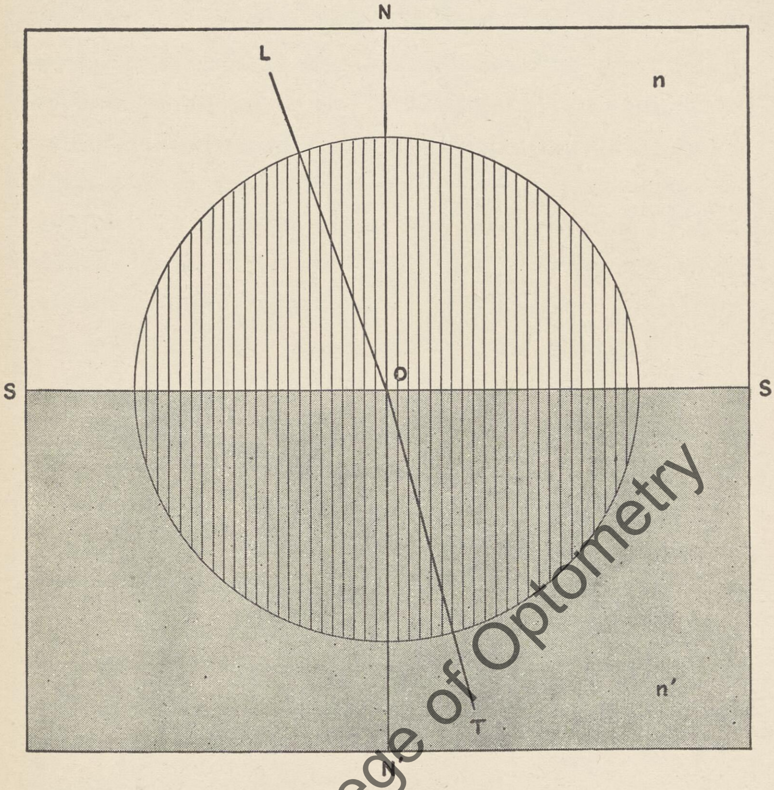


Figure 19.

vertical and horizontal diameters, NN′ and SS. Fine vertical lines divide the horizontal diameter into 24 equal parts. Two clock hands are arranged to rotate freely and independently around O. SS represents a bounding surface separating two media, n and n′.

If L is an incident ray striking the surface SS at O, T can be made to show the direction this ray will take in the denser medium n′ after being refracted. For instance, if n is air and n′ is water, the index of refraction for water being $4/3$, the sine of the angle of incidence must be 4 while the sine of the angle of refraction is 3. If L be considered the incident ray and the angle LON is such that L cuts the circle at the 4th division, the sine of the angle LON is equal to $4/24$ or .166; so that in this case the refracted ray will form an angle N′OT whose sine is $3/24$ or .125. If L is swung around to cover 8 divisions the sine of the angle LON will be $8/24$ or .333, and T should then be made to cover 6 divisions, so that the sine of the angle N′OT is $6/24$ or .25.

Continuing in this way, if L is swung around to coincide with SS it will form an angle of 90 degrees with N, covering all 24 divisions, and its value will be $24/24$ or 1.00—the sine of a 90 degree angle. The refracted ray must now form such an angle with N′ that its sine is .75 or $18/24$, that is, T must cut the circle 18 points from N′.

If, now, the direction of the light is reversed, making T the incident and L the refracted ray, in the case where sine LON equals 1.00, and the angle being 90 degrees, the refracted ray emerges parallel to the surface—the angle TON′, whose sine is .75, is the critical angle for water. If the angle TON′

is greater than .75 the ray cannot leave the water and it is totally reflected internally.

If the critical angle is designated by x

$$n = \frac{\sin 90}{\sin x}$$

whence

$$\sin x = \frac{1}{n}$$

CHAPTER IV.

REFLECTION AT SPHERICAL SURFACES.

Convex or concave polished surfaces that are a portion of the surface of a sphere form spherical mirrors. The direction of the reflected rays from a curved surface may be determined geometrically according to the laws of reflection.

The center of the hollow sphere, of which the mirror forms a part, is called the *center of curvature* or the *center* of the mirror. The middle point of the reflecting surface, which must not be confused with the center of curvature, is called the *vertex.* A line passing through the center of curvature and the vertex is the *principal axis;* all other lines that pass through the center of curvature are *secondary axes.* When an object-point is situated on the principal axis its image is formed on the principal axis; when the object-point lies on a secondary axis its image will then be formed on this secondary axis. Since an object can have only one point on the principal axis, every other point must lie on a secondary axis. The axes are radii of curvature and, therefore, normals to the reflecting surface.

If a small pencil of incident rays is parallel to the axis of a spherical mirror the point on the axis in which these rays meet or appear to meet again,

after being reflected, is called the *principal focus*. The distance from the principal focus to the mirror is called the *principal focal distance* of the mirror, and this distance is equal to one-half the radius.

In order that a reflector should produce a distinct image of an object placed before it, it is necessary that the rays diverging from each point of the object should, after reflection, diverge from or converge to some common point.

Rays that diverge from an object-point on the axis will, after reflection by a spherical mirror, be made to meet again, or to appear to meet again, in another point on the axis. This second point is the image-point, and it and the object-point are called *conjugate foci,* because if the direction of the rays is reversed, that is, incident from the image-point, they will meet again in the original position of the object-point. The two points are interchangeable. The distance from the object to the mirror is called the *first conjugate focal distance*, and that from the image to the mirror is called the *second conjugate focal distance.*

In the case of a plane reflecting surface it has been shown that the image is equal and in all respects similar to the object, but this does not occur in the case of a spherical mirror. The two classes of spherical reflectors, concave and convex, will be explained in succession.

Reflection of a Ray of Light by a Spherical Surface. If a ray of light proceed from an object-point, L (Figure 20), and fall at O upon a spherical concave mirror, OS, whose center is C and be reflected at O so as to intersect the radius CS in the point L′, the distances LO and L′O will then be to each other in the same ratio as the distances LC and L′C. For, since the radius CO is perpendicular to the surface, the angles i and r are the angles of incidence and reflection and therefore equal. In the triangle LOL′, since OC bisects the vertical angle, it will divide the base LL′ into segments proportional to the sides; therefore

$$LO : L'O :: LC : L'C \quad . \quad . \quad . \quad (1)$$

The distances of the points L and L′ from the surface are to each other in the ratio of their distances from the center.

L and L′ are conjugate foci and, therefore, if the ray of light proceed from L′ it would be reflected back to L. It is important to know that if any ray of light be turned back in its course it will describe the same path as before, but in the opposite direction.

If the ray of light were to fall on a convex mirror as in Figure 21 the same proof would apply as before, with this difference, that the radius CO bisects the external angle of the triangle LOL′; but in this case also, it is easy to show that

$$LO : L'O :: LC : L'C.$$

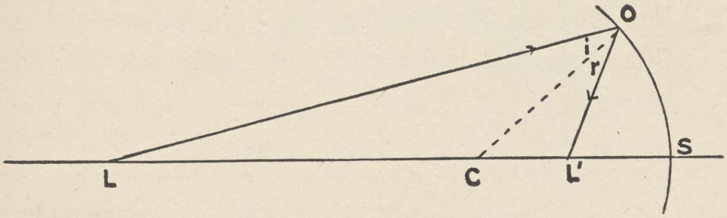


Figure 20.

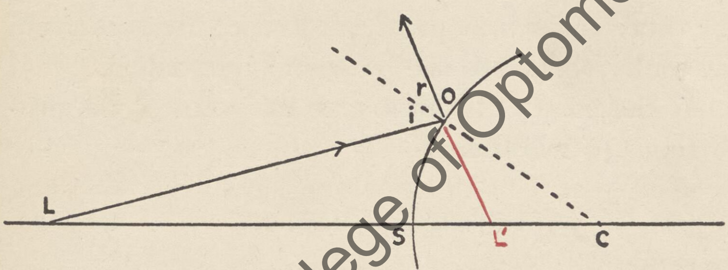


Figure 21.

If the incident ray LO is parallel to the radius CS, as in Figure 22, then LOC is equal to COL′, because they are the angles of incidence and reflection, and L′OC is equal to OCL′; therefore OL′C is an isosceles triangle, and L′O is equal to L′C.

Hence, if an incident ray of light is parallel to a radius of a spherical mirror, the reflected ray will

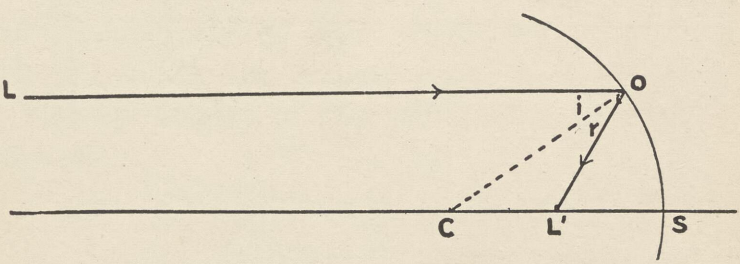


Figure 22.

cut that radius in a point L′, whose distances from the center, C, and from the point O are equal.

If the point O be very near the point S, the proportion (1) will become

$$LS : L'S :: LC : L'C \quad . \quad . \quad . \quad (2)$$

Reflection of a Pencil of Rays from a Spherical Mirror. When a pencil of rays diverging from an object-point (L, Figure 23) falls upon a spherical mirror, each ray, LO′, LO″, LO‴, will be reflected and will intersect the line LS in a point L′, L″, L‴,

and the position of each point will be found for each ray from the proportion (1). It will be different for each ray, because it depends on the line LO which enters into the proportion.

Only the rays which traverse the same circle, having S as the center, will be imaged in the same point on the axis. Each circle of the surface forms an image-point and these images are situated along

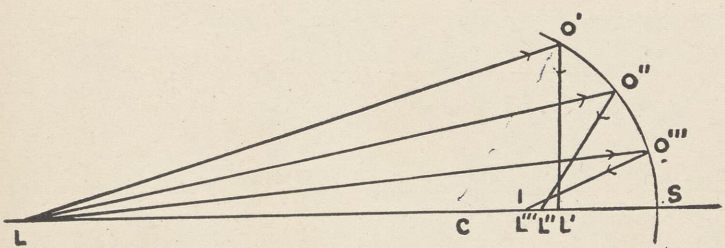
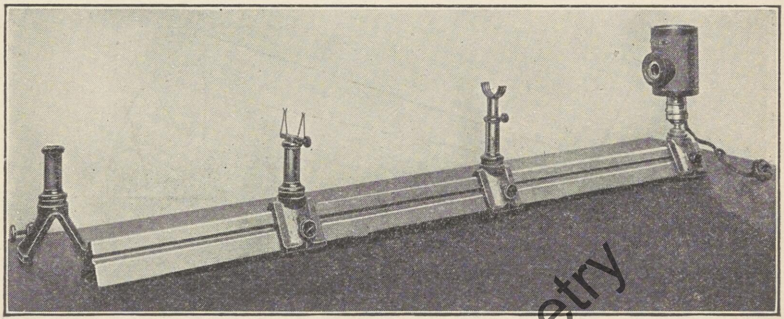


Figure 23.

the axis, forming a *focal line*. Where these rays cross one another a bright line is formed which is called a *caustic*.

The lines LO will be nearly perpendicular to the mirror in the neighborhood of the point S and will, therefore, be nearly equal. Consequently, the points L′, L″, L‴, found from (1) for such rays, will be crowded together and produce a bright light or focus. The position of this focus is found by proportion (2), in which LO and L′O become LS and L′S. It will be assumed that all the rays are very close to the principal axis of the mirror.

Optical Bench and Fittings. An optical bench of some kind is indispensable for the performance of many of the experiments that follow. An optical bench for teaching must of necessity be so constructed that, while it enables accurate work to be done, it can be easily and quickly set up, and at the same time be substantial enough to stand real hard usage. The ordinary stock equipments offered



A B C D E

Figure 24.

by dealers do not seem to answer these requirements; moreover, in very few of these can trial-case lenses and accessories be easily used—a disadvantage and an added expense in an institution where trial cases are always at hand.

The bench, B (Figure 24), was obtained from Carl Zeiss. It is made of triangular steel, one meter in length, having a groove along each side to allow the saddle-stands to be securely fixed. Removable bolts and nuts are supplied with the bench so that it may, if desired, be fastened to a table or other base; but

its weight makes this unnecessary and sometimes inconvenient. The saddle-stands, as shown at A, were also obtained from Zeiss; the modifications and carriers were made by Wall & Ochs.

Each saddle-stand is provided with two set-screws, one to fix into the groove and hold it in position on the bench, and the other to hold the carrier. It is so formed that it will stand and may be lifted off the bench and placed on the table—a very necessary proceeding in some experiments. As the bench is not scaled, we have had a white line etched at the center of one side of each saddle-stand, so that direct measurements can be made with a meter stick. Measuring by this method is easier than if the bench itself were ruled.

It will be seen in the illustration that the saddle-stand at E has had the entire upper portion removed and a lamp socket, with switch and cord attachment, set in its place. This holds a round, frosted 25 watt lamp, housed in a Thorington chimney which is provided with an iris diaphragm. In front of the diaphragm-opening there was placed a single trial-lens cell of standard size.

The carrier in D is a three-cell compartment to hold regulation trial-case rings. It is set in a standard having a collar so that, when dropped into the saddle-stand and a lens placed in it, the lens will be exactly centered with the aperture in the chimney.

The stand C contains a carrier consisting of two adjustable brass clips, mounted on a slotted cross

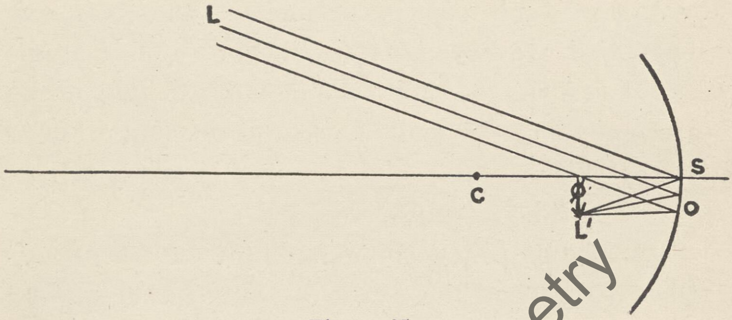


Figure 25.

piece. The upright, which fits easily into the saddle-stand, may be raised to the desired position and fixed by the set-screw in the stand. While this form of carrier can, when needed, be used for trial-lenses, it is intended for holding screens, stops and other articles for which the cells in carrier D are unfitted.

EXPERIMENT *to find the Principal Focus of a Concave Mirror.* Adjust a concave mirror on the optical bench so that the axis of the mirror is parallel to the bar. Direct the mirror to a well illuminated distant object. In front of the mirror arrange an image-screen, being careful that it does not obstruct the light, and find the position where a clear image of the distant object is formed. If the image-screen is of thin ground-glass the image can be seen from either side. Then, without changing the position of either carrier, replace the screen with a tall pin or wire and verify the position of the image by parallax. Measure the distance from the mirror to the image. This is the principal focal distance, F, of the mirror.

Since, in general for any parallel ray, L′S equals L′C when the point O approaches very near the point S, the principal focus (Φ, Figure 25) will lie at the point of bisection of the radius CS. The image is real and inverted. If either L or L′ coincides with Φ, the other must be at an infinite distance.

The proportions (1) and (2) may be replaced by an algebraic formula as follows:

Let OP be drawn perpendicular to CS, as in Figure 26, then since

$$OL'S = OCS + COL'$$

and

$$OLS = OCS - COL$$

adding together we find, because $COL' = COL$,

$$OL'S + OLS = 2\ OCS$$

But from the proportions of right-angled triangles we have

$$\sin OL'S = \frac{OP}{OL'}$$

$$\sin OLS = \frac{OP}{OL}$$

$$\sin OCS = \frac{OP}{OC}$$

and since, if O be very near S, the angles above will be very small and be equal to their sines, we find

$$\frac{OP}{OL} + \frac{OP}{OL'} = 2\frac{OP}{OC}$$

and writing $OL = f$, $OL' = f'$ and $OC = R$, we finally obtain, dividing both sides of the equation by OP,

$$\frac{1}{f} + \frac{1}{f'} = \frac{2}{R} \quad . \quad . \quad . \quad . \quad (3)$$

In this equation we have three quantities: the distance of L′ from the mirror, the distance of L from the mirror, and the radius of the mirror; any two of which being given, the other can be found.

When a pencil of light, after reflection, is convergent it is given the positive sign (+) because the rays meet in a real focus in front of the mirror, and they can be imaged on a screen. When the pencil is divergent, after reflection, its focus is found by

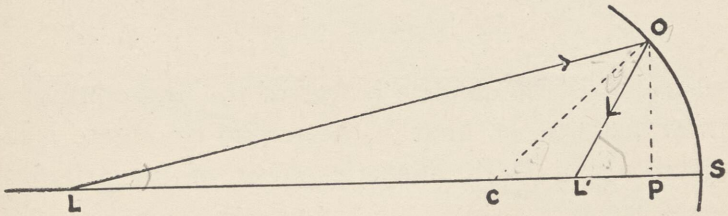


Figure 26.

projecting the rays backward until they meet in a virtual image behind the mirror, and in this case the negative (—) sign is given. The opposite terms are used for convergent and divergent incident rays. In the diagrams the red lines indicate negative rays. Concave mirrors are given the positive (+) sign and convex mirrors are given the negative (—) sign.

The focus L′ is called the *conjugate* to the focus L. If L lies at infinity, and the incident ray is parallel to the axis of the mirror, L′ will cut the axis ½ the distance of C from the mirror. This point is the principal focus of the mirror and is

designated by the sign Φ. The principal focal distance will be designated by F. Since F equals ½ R, equation (3) may be written

$$\frac{1}{f} + \frac{1}{f'} = \frac{1}{F} \quad . \quad . \quad . \quad . \quad (4)$$

If the distances f, f′ and F are measured in meters the reciprocals of these distances will represent diopters, and formula (4) may be written

$$D' + D'' = D \quad . \quad . \quad . \quad . \quad (5)$$

where D′ is the dioptric power of the first conjugate focal distance, D″ that of the second conjugate focal distance and D the dioptric power of the principal focal distance.

Example. An object is placed at a distance of 100 centimeters in front of a concave mirror, and its image is formed at a distance of 25 centimeters in front of the mirror. What is the principal focal distance of the mirror?

Substituting these figures in formula (4) we write

$$\frac{1}{100} + \frac{1}{25} = \frac{1}{20}$$

The principal focal distance, F, is 20 centimeters. Using formula (5), and substituting the equivalent dioptric powers for these distances,

$$1\ D + 4\ D = 5\ D$$

Experiment *to find the Conjugate Foci of a Concave Mirror.* Draw an arrow or other figure on a ground-glass disc and place it in the opening of the Thorington chimney. Place this luminous object, which is represented by L, Figure 27, on one end of the optical bench. Facing it, on the other end, place a concave mirror (radius of about 25 cm.) in a suitable carrier so that it is in line with and faces the object. On another carrier erect an

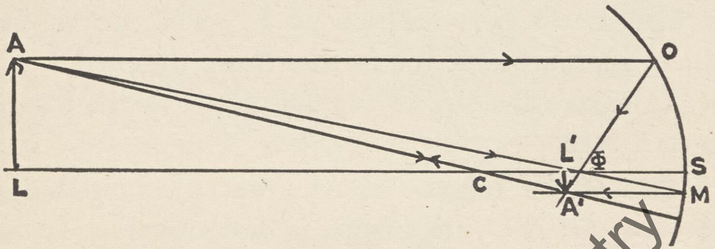


Figure 27.

image screen, being careful not to obstruct the light, and slide the saddle-stand along the bench until L′ is found, where a clear image will be formed. Measure the distances of the object and its image, f and f′, from the mirror and prove formulæ (4) and (5).

By construction it will be seen that the ray AO being parallel to the axis will, after reflection, be directed through the principal focus Φ; AΦ will be reflected as if it came from Φ and emerge parallel to the axis; AC, passing through C, will be reflected backward along its original path, as are LS and every other ray passing through the center of curvature.

The point A′, where the reflected rays cross, is the image of A.

The image of L will be formed on the principal axis where a perpendicular is dropped from A′. Every other point in the object between L and A will have its image formed between L′ and A′, therefore L′A′ is the image of LA.

The image is real and inverted. L and L′ are conjugate to each other and if their positions be changed so that L is placed at L′ an image will be formed at the first position of L. The nearer the object approaches Φ, the farther away the image will be. Measure the size of the object and of the image in each case.

Size of the Image. If AL, the object, is designated by O; A′L′, the image, is designated by I; the distance from L to Φ designated by l, and from L′ to Φ by l', the triangles ALΦ and MΦS or OΦS and A′ΦL′ give us the relations

$$\frac{O}{I} = \frac{l}{F} = \frac{F}{l'}$$

or

$$ll' = FF \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Since F is equal to ½ the radius,

$$\frac{O}{I} = \frac{l}{F}$$

can also be written, if R signify the radius,

$$\frac{O}{I} = \frac{2l}{R}$$

or

$$R = \frac{2lI}{O}$$

which is the formula used in ophthalmometry.

EXPERIMENT. *Reflection by a Concave Mirror when the Object is at the Center.* Arrange the ob-

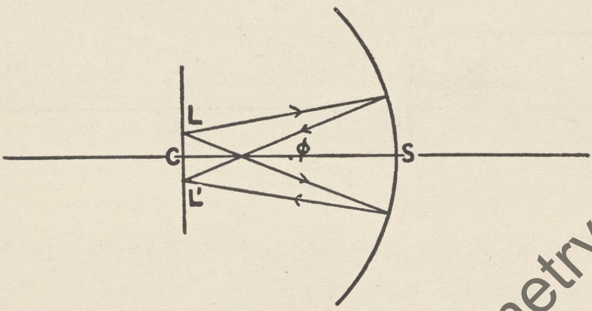


Figure 28.

ject and the image-screen so that they are at the same distance from the mirror, as is shown in Figure 28. If the image and the object had perfectly coincided each ray of light must have returned exactly along its path and have struck the surface normally. This point is the center of curvature of the mirror and it lies twice the focal distance from the mirror, because then $^1/f = {}^1/R$. The image is real, inverted and of the same size as the object.

Experiment *to find the Virtual Image by a Concave Mirror.* Erect a short pin in front of a concave mirror, but at a distance less than F. With another pin behind the mirror, and tall enough to be seen over the top, locate by parallax the position of the image of the first pin. Measure the distances and prove equations (4) and (5). In this

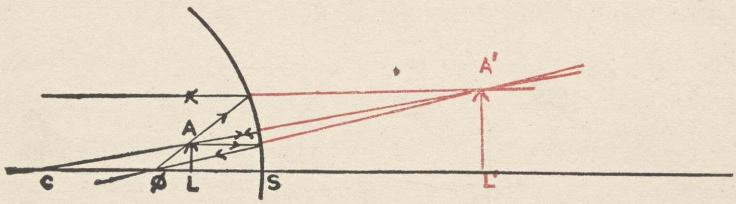


Figure 29.

case the sign for the second conjugate focal distance is negative (— f′)

$$\frac{1}{f} + (-\frac{1}{f'}) = \frac{1}{F}$$

and

$$D' + (-D'') = D$$

Figure 29 shows the method of finding the virtual image at a concave mirror by construction. The reflected rays, being divergent, are made to meet by projecting them to a point behind the mirror where a virtual, erect and large image is formed.

Example. An object is placed 10 centimeters in front of a concave mirror; a virtual image is formed

at a distance of 20 centimeters behind the mirror. What is the focal distance of the mirror?

By formula (4) we have

$$\frac{1}{10} + (-\frac{1}{20}) = +\frac{1}{20}$$

It is a concave mirror (because it is positive) having a focal distance of 20 centimeters, and the radius of curvature, therefore, is 40 centimeters.

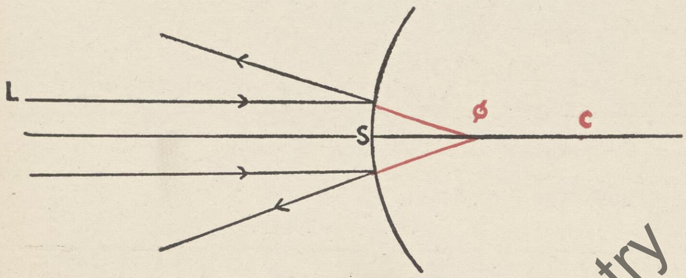


Figure 30.

By formula (5) we write

$$+ 10\text{ D} - 5\text{ D} = + 5\text{ D}$$

Experiment *to find the Principal Focus of a Convex Mirror.* The center of curvature of a convex mirror lies behind the reflecting surface, at C in Figure 30, and the principal focus, Φ, lies midway between the center of curvature and the surface, S. The principal focal distance, F, is given the negative sign.

Direct a convex mirror toward a distant object and look into the mirror from the same side. A

small, virtual and erect image will be seen. Locate the position of the image by the method of parallax.

Experiment *to find the Conjugate Foci of a Convex Mirror.* Set up a convex mirror on the optical bench and, for the object, fix a pin at a distance in front of the mirror. Adjust a tall pin behind the mirror so that it coincides by parallax with the image of the first pin as seen in the mirror.

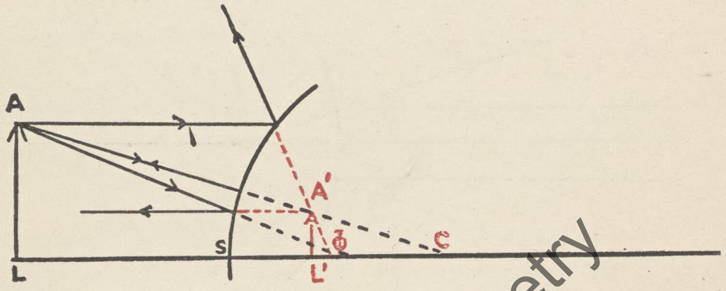


Figure 31.

The image is small, virtual and erect. Measure the distances and calculate by equations (4) and (5), f′ being negative. Find the size of the image by the formula for ophthalmometry. Figure 31 shows how the conjugate foci of a convex mirror may be found by construction.

Example. An object is placed at a distance of 20 centimeters in front of a convex mirror. An image is formed at a distance of 10 centimeters behind the mirror. What is the principal focal distance of the mirror?

In this case f is positive and f′ is negative. By formula (4) we write

$-\frac{1}{y} = -\frac{1}{10} = \frac{1}{20}$

$$\frac{1}{20} + (-\frac{1}{10}) = -\frac{1}{20}$$

$-20 = -2y + y$

$= -y$

and since the principal focal distance, F, is —20 centimeters the mirror must be convex.

By formula (5) we find the power

$$5\ D - 10\ D = -5\ D.$$

If F and f are given we can find f′. Suppose we have a convex mirror, F = —50 centimeters, and an object placed at a distance, f, of 25 centimeters; then

$$\frac{1}{f} + \frac{1}{f'} = -\frac{1}{F}$$

and substituting the distances

$$\frac{1}{25} + (-\frac{1}{16.6}) = -\frac{1}{50}$$

or, by formula (5),

$$4\ D - 6\ D = -2\ D.$$

Suppose the object is 6 centimeters in size; find the size of the image by the formula for ophthalmometry (page 59).

$$\frac{2lI}{O} = R$$

In this case R = 2F = 2 × 50 = 100 centimeters, and l = F + f = 50 + 25 = 75 centimeters. Replacing the signs by these figures

$$\frac{2 \times 75 \times 4}{6} = 100$$

the image is 4 centimeters in size.

EXPERIMENT *to find the Center of Curvature of a Convex Mirror with the aid of a Convex Lens.* Set

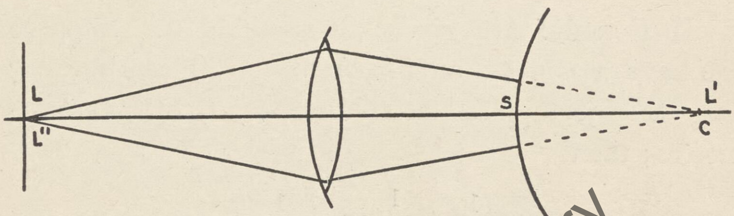


Figure 32.

up a convex lens on the optical bench and find the place where is formed the image of an illuminated object. Interpose a convex mirror between L′ and the lens (Figure 32). Adjust the mirror so that the image is formed on the object-screen beside the illuminated opening. When this is the case the light, being returned along the original path, must strike the mirror normally, and the rays, being radii of the surface, would, if prolonged, meet at the center, C.

CHAPTER V.

REFRACTION BY A SPHERICAL SURFACE.

Let the spherical surface, SH, in Figure 33, separate two media, the rarer of which is designated by n and the denser by n′. The center of curvature is C. The angle of incidence for the ray LO‴ is

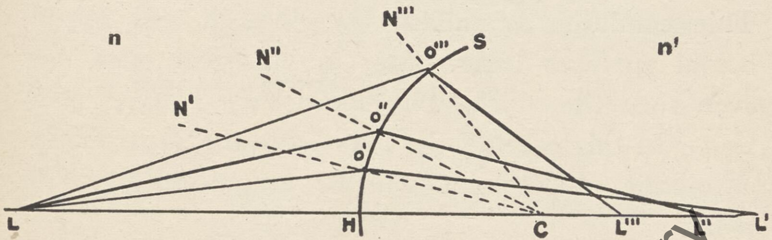


Figure 33.

LO‴N‴ and the ray is deviated to L‴, making CO‴L‴ the angle of refraction. The ray LO″, whose angle of incidence is less than LO‴, is deviated to L″, and LO′ is deviated still less, meeting the principal axis at L′.

The ray LH coincides with the principal axis and passes through without being refracted. All the rays that traverse the same circle, having H as center, are united at the same point on the axis, providing the light is monochromatic. Sunlight, which is composed of all the colors, is decomposed by refraction so that even the rays in the same

circle are more refracted as they approach the violet end of the spectrum. This is called *chromatic aberration.*

The points where the rays cross each other in being directed to their respective focal points, L′, L″, L‴, are distinguished by their greater luminosity and their union forms what is called a caustic.

In order for homocentric rays to remain homocentric the surface must have such a form that the angles of incidence will be everywhere the same. This condition is fulfilled in ellipsoid and hyperboloid surfaces better than in spherical ones, but even then the object-point must have a fixed position. In this case all the rays from the same monochromatic object-point will be united at a single point.

In the sphere each circle of the surface forms an image of the object-point and all the images are situated along the axis, forming the focal line, as was described under Reflection. In the paraboloid of revolution all the circles form their images at the same point. We shall assume, as was done in reflection, that only rays very near the axis are allowed to pass through the refracting surface and that the image of a point is a point.

Principal Foci. When the object is at infinity, the rays emitted are considered parallel and the point behind the refracting surface where these rays meet is called the posterior or second principal focus and

will be designated by Φ′ (Figure 34). The distance from the surface, H, to Φ′ is the second or posterior principal focal distance, designated by F′. If an object-point is placed at Φ′ the emergent rays will be parallel after refraction at the surface.

When the rays are parallel in the second medium the point Φ, where these rays meet in front of the

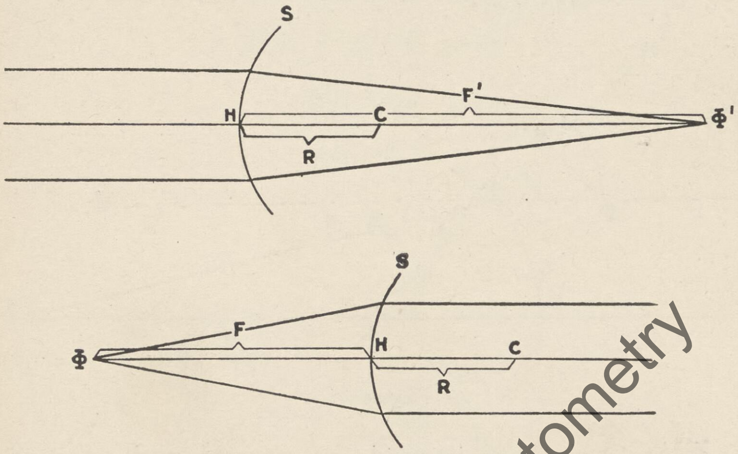


Figure 34.

refracting surface, is called the anterior or first principal focus. The distance from H to Φ is the first or anterior principal focal distance and is designated by F. If an object-point is placed at Φ the emergent rays will be parallel in the second medium.

The point C is the center of curvature of the sphere of which the surface is part. The distance CH is called the radius of curvature and is represented by R. The line passing through the vertex

of the surface and C is the principal axis of the system. All other radii of curvature are secondary axes and a ray coinciding with one of these, that is, passing through the center of curvature, continues in a straight line.

Let Figure 35 represent a spherical curved surface separating a medium, n, from a denser medium, n′. The ray, L, parallel to the axis, strikes the sur-

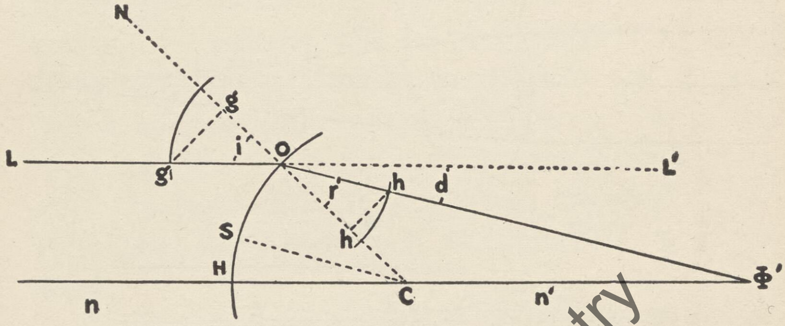


Figure 35.

face at O, is bent toward the normal and directed to the posterior principal focus, Φ′.

The angle of incidence, i, is formed by the incident ray, LO, with the normal ON; the angle of refraction, r, is formed by the refracted ray, OΦ′, and the normal OC; the ray is deviated through the angle L′OΦ′ and

$$d = i - r$$

where d signifies the angle of deviation.

Draw SC parallel to OΦ′, and we have the angle SCO to correspond with the arc OS, and SCH to

correspond with the arc SH. If the angles are very small we may say that

$$\frac{\Phi'H}{CH} = \frac{\text{arc HO}}{\text{arc HS}}$$

and since the angle

$$OCH = NOL = i$$

$$SCH = L'O\Phi' = d$$

$$\Phi'H = F'$$

$$CH = R$$

$$\frac{F'}{R} = \frac{i}{i - r}$$

The angles being small and proportional to their sines, we may say

$$\frac{F'}{R} = \frac{\sin i}{\sin i - \sin r} \quad . \quad . \quad . \quad . \quad (a)$$

If we describe a circle with O as center, then gg is the sine of the angle of incidence, hh the sine of the angle of refraction and, for light entering a denser medium, we have the proportion (page 30)

$$\frac{\sin i}{\sin r} = \frac{n'}{n}$$

That is,

$$n \times \sin i = n' \times \sin r$$

and substituting n for sin r and n′ for sin i in (a)

$$\frac{F'}{R} = \frac{n'}{n' - n}$$

or

$$F' = \frac{n' R}{n' - n} \quad . \quad . \quad . \quad (b)$$

For the anterior principal focal distance the light is incident in the denser medium, n′, and LOC (Figure 36) is now the angle of incidence, i, and ΦON is the angle of refraction, r. The ray LO,

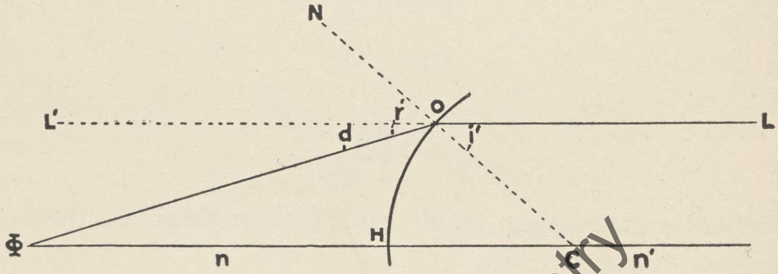


Figure 36.

parallel to the axis, strikes the surface at O, is bent away from the normal, and is directed to the anterior principal focus, Φ, in the rarer medium, n. The ray is deviated through the angle L′OΦ, so that in this case

$$d = r - i$$

In the two triangles HCO and HΦO, the side OH being common, and substituting the arc for the sine, the angles are inversely proportional to the bases of their respective triangles. If the angles are very small and O is very close to H, we may say

$$\frac{H\Phi}{HC} = \frac{OCH}{O\Phi H} \quad . \quad . \quad . \quad (c)$$

The distance $H\Phi$ is designated by F, HC by R, the angle OCH = LOC = i, $O\Phi H = L'O\Phi = d$; therefore (c) may be written

$$\frac{F}{R} = \frac{i}{r - i}$$

If the angles are small we may substitute the sines, and this may be written

$$\frac{F}{R} = \frac{\sin i}{\sin r - \sin i} \quad . \quad . \quad . \quad . \quad (d)$$

For light incident in the denser and emergent in the rarer medium, we have

$$\frac{\sin i}{\sin r} = \frac{n}{n'}$$

or

$$n \times \sin r = n' \times \sin i$$

which is the inverse of that when light is emergent in the denser medium.

Substituting n for sin i, and n′ for sin r in (d),

$$\frac{R}{F} = \frac{n}{n' - n}$$

or

$$F = \frac{nR}{n' - n} \quad . \quad . \quad . \quad . \quad . \quad (e)$$

If the first medium is air and $n = 1$, and if n' be designated by n, we may write for (e)

$$F = \frac{R}{n - 1} \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(I)}$$

and for (b)

$$F' = \frac{nR}{n - 1} \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(II)}$$

When the distances are measured in meters, since the dioptric power is inversely proportional to the focal distance measured in meters, we have, denoting the anterior dioptric power by D,

$$D = \frac{n - 1}{R}$$

and for the posterior dioptric power, designated by D′,

$$D' = \frac{n - 1}{nR}$$

The difference between F and F′ is equal to the radius. There is another relationship,

$$\frac{F'}{F} = \frac{\dfrac{n'R}{n' - n}}{\dfrac{nR}{n' - n}} = \frac{n'}{n}$$

That is to say that the principal focal distances are to each other as the indices of refraction of the

media to which they belong. When n is equal to 1, *i.e.,* for air, we obtain

$$\frac{F'}{F} = n$$

or

$$F' = F \times n$$

The posterior principal focal distance is equal to the anterior principal focal distance multiplied by the index of refraction of the refracting medium. The posterior dioptric power is equal to the anterior dioptric power divided by the index of refraction—

$$D' = \frac{D}{n}$$

In formula (I) it will be seen that, in order to find F, we merely divide the radius of curvature of the refracting surface by the index of the denser medium less 1. Having found F it is easy to find F′ and the dioptric powers.

Example. A convex surface, having a radius of curvature of 8 millimeters, separates air from a substance having an index of refraction of 1.336. Substituting these figures in formula (I) we have

$$F = \frac{8}{1.336 - 1} = \frac{8}{.336} = 23.8 \text{ mm.}$$

and $F' = 23.8 \text{ mm.} \times 1.336 = 31.7968 \text{ mm.}$

Suppose this same surface separates air from water, which has an index of refraction of 1.33 or $\frac{4}{3}$. When F is 3, F′ is 4, that is, F′/F equals $\frac{4}{3}$ or 1.33. The difference between the two is the radius of the refracting surface or 8 mm. Now in order to obtain two numbers which shall be to each other as 4 is to 3 and whose difference is 8, it is only necessary to multiply 4 and 3 by 8. Hence F′ will be 4×8 or 32 mm., and F will equal 3×8 or 24 mm.

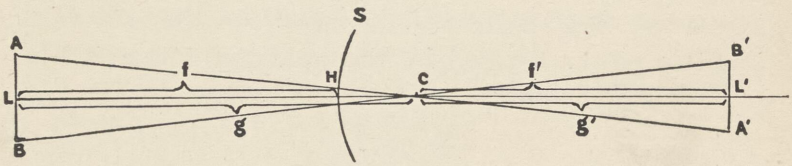


Figure 37.

If the medium is glass, with an index of refraction of 1.5 or $\frac{3}{2}$, and has the same radius, then $F' = 3 \times 8$ or 24 mm. and $F = 2 \times 8$ or 16 mm.

Image formed by a Single Refracting Surface. An object-point at L (Figure 37), being situated on the principal axis, will form an image somewhere on the principal axis. Another object-point at A should form its image on the secondary axis which passes through A and the center of curvature, C, and at the same distance behind the latter, since L and A are considered to be the same distance from S. Hence we have only to draw a straight line from A through C and mark upon it the length f′

from the point where this line cuts the surface in order to find the point A′, which will be the image-point of A. The same takes place for B and for all points between A and B.

Each object-point gives off an infinite number of rays, forming a cone, and Figure 38 illustrates how the lines AA′, LL′ and BB′ become the axes of cones of divergent rays which, after refraction, unite and form the image-points.

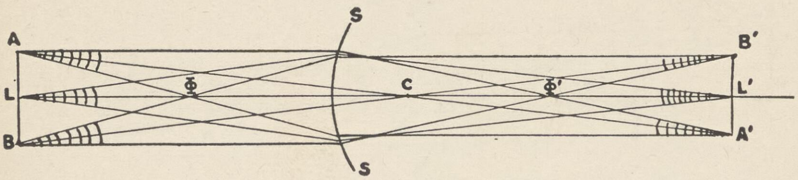


Figure 38.

The image is inverted relative to the object and geometrically similar to it. AB and A′B′ are corresponding sides of two similar triangles whose angles are equal. Hence to find the size of the image we write

$$\frac{A'B'}{AB} = \frac{L'C}{LC}$$

Let the object be designated by O, the image by I, LC by g and L′C by g′, we have

$$\frac{I}{0} = \frac{g'}{g} \text{ or } I = \frac{0g'}{g} \quad . \quad . \quad . \quad . \quad (a)$$

In order to find, by construction, the image formed by a spherical surface we have only to draw certain lines of direction from the extremities of the object, bearing in mind their direction after being refracted.

Let LA (Figure 39) be the extremities of an object, designated by O, placed before a spherical surface SS. If we can find the image-points of the extremities of the object we shall have the extremities

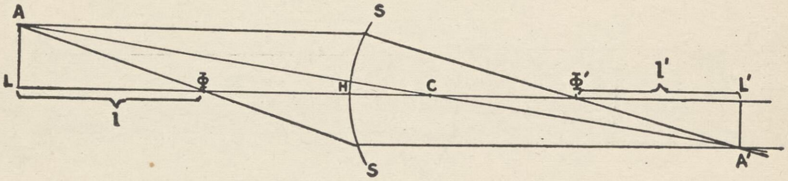


Figure 39.

of the image. Since L lies on the principal axis its image will be formed somewhere on the principal axis, and we have only to find where the image of A is formed.

One line of direction representing a ray from A parallel to the axis will, after refraction, be made to pass through the posterior principal focus, Φ'. Another ray, passing through the center of curvature, C, will continue in a straight line. The point A', where these lines cross, is the image of A. Still another line of direction, passing through the anterior focus, Φ, will be parallel to the axis, after refraction, and meet the other lines in A'. A perpendicular

dropped to the axis at A′ will cut the axis in L′ which is the image of L.

Designate the object by O, the image by I, the distance from L to Φ by l, and from L′ to Φ′ by l'. The triangles AΦL and HΦS, SΦ′H and L′Φ′A′ give us

$$\frac{0}{\mathrm{I}} = \frac{l}{\mathrm{F}} = \frac{\mathrm{F}'}{l'}$$

from which we derive

$$ll' = \mathrm{FF}' \quad . \quad . \quad . \quad . \quad . \quad (\mathrm{III})$$

a most useful formula for the conjugate foci.

We call the object-point and the image-point, L and L′, *conjugate foci,* and the distances f and f′, which separate these two points from the refracting surface, *conjugate focal distances.* One is commonly called the first or anterior conjugate focal distance and the other the second or posterior conjugate focal distance.

When the object-point L lies at a finite distance in front of Φ, L′ should be beyond Φ′, and f and f′ are positive. The plus sign is given to f and f′ as long as the object and image are on opposite sides of the bounding surface. If the image and object are on the same side of the refracting surface, f or f′ takes the negative sign.

Since

$$l = \mathrm{f} - \mathrm{F}$$

and

$$l' = \mathrm{f}' - \mathrm{F}'$$

l is positive as long as f is greater than F, and negative when F is greater than f. The same is true for l' in respect to the relative distances of f′ and F′. When l or l' is positive it is measured from Φ or Φ′ on the side away from the surface; when they are negative they are measured from Φ and Φ′ towards the surface.

Example. L being 250 mm. in front of a convex spherical surface having a radius of curvature of 5 mm., F and F′ being respectively 15 and 20 mm., by formula (III) we have, since $l = 250 - 15 = 235$,

$$FF' = 15 \times 20 = 300$$

$$ll' = 235 \times 1.2 = 300$$

Therefore, if $l' = 1.2$ mm., L′ is situated $F' + l' = 20 + 1.2 = 21.2$ mm. behind the surface. If the object-point is situated at this point in the second medium it will produce its image 250 mm. in front of the surface.

If the luminous point is brought nearer than Φ the rays must diverge after refraction. Suppose L lies at a point 10 mm. in front of the surface. Then $l = -5$, and with the same surface we have

$$-5 \times -60 = 300$$

Therefore $l' = -60$ and

$$f' - F' = l' = -40 - 20 = -60 \text{ mm.}$$

and —40 for f′ indicates that L′ is situated on the same side as L, that is, in the air, 40 mm. in front of the surface. When the object-point is on the surface itself the image and the object coincide.

For the size of the image we may say

$$I = \frac{OF}{l}$$

and . . . . . (IV)

$$I = \frac{Ol'}{F'}$$

Example. Size of the object, O, 12 mm.; distance of the object from the surface, f, 1000 mm.; radius of curvature of the surface, R, 5 mm.; posterior principal focal distance, F′, 20 mm.; anterior principal focal distance, F, 15 mm.

By formula (III), knowing that $l = 1000 - 15 = 985$ mm., we find that $l' = .304$ mm. and the image is formed at a distance $F' + l' = f' = 20$ mm. + .304 mm. = 20.304 mm. behind the surface. We now have the size of the object, the values of F, F′, l and l'. The size of the image can be found by formulæ (IV),

$$I = \frac{12 \times 15}{985} = .18 \text{ mm.}$$

or

$$I = \frac{12 \times .304}{20} = .18 \text{ mm.}$$

and by formula (a, page 75).

$$I = \frac{12 \times 15.304}{1005} = .18 \text{ mm.}$$

When the rays are incident at Φ they are parallel to each other after refraction. Rays from an object closer than F are divergent after refraction and cannot meet; they form a virtual image by being prolonged backward as in Figure 40. Calculation may be made from formulæ (III) and (IV) by using the negative sign properly.

The image increases in size and recedes from the surface in proportion as the object approaches.

When the object lies in the second medium, at a distance closer to the surface than F′, the same construction can be made as in Figure 40. The image will be virtual and is formed by producing the rays backward in the same manner.

When the rays are rendered convergent, as in Figure 41, f is then negative because the object-point is situated at L where the convergent rays would meet if produced; that is, virtually behind the surface on the same side as the image. In order to find L′ it is necessary to give the negative sign to f.

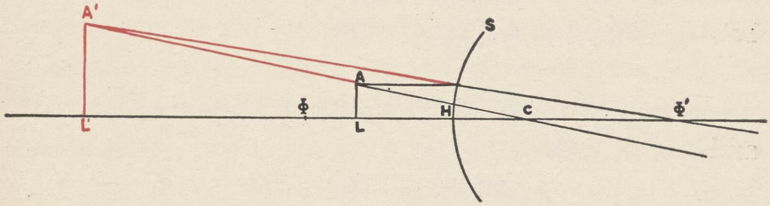


Figure 40.

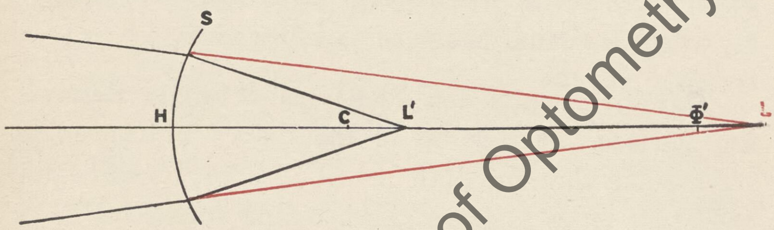


Figure 41.

6

Example. Let F′ equal 20 mm., F equal 15 mm. and suppose that the rays which strike the surface converge to a point L, situated 250 mm. behind the surface; f is then minus and equals —250 mm. The formula $ll' = FF'$ being used, we have $l = -250 - 15 = -265$ and we know that

$$FF' = 300$$

and

$$-265 \times -1.13 = 300$$

Having found l' to be —1.13, f′ must therefore be 1.13 mm. less than the posterior principal focal distance (20 mm.), or 18.8 mm.

Invert the case and make L′ the object. The rays, after refraction, diverge as if they came from L, where a virtual image of L′ is formed.

Cardinal Points and Planes of a Single Refracting Surface. According to the theory of Gauss, if we assume that the light pencils are very small, and therefore close to the axis, a single refracting surface becomes a system consisting of 4 cardinal points, all situated on the principal axis, and 3 planes, perpendicular to the axis, at 3 of these points (Figure 42).

1. Principal Point, H. The principal point is situated at the intersection of the bounding surface with the optic axis. A plane passed through H, perpendicular to the optic axis, is the *principal*

plane. The plane, then, takes the place of the curved surface, and from it are measured the principal and conjugate focal distances.

2. First Principal Focus, Φ. All rays emanating from the first principal focus are parallel to each other and to the principal axis after having passed through the surface. Parallel rays incident in the second medium are brought to a focus in this point. A plane passed through Φ perpendicular to the axis is called the *first principal focal plane*. Rays of light diverging from any point situated on the first principal focal plane will, after refraction, be parallel to each other and to the secondary axis from which they emerge.

3. Second Principal Focus, Φ′. The second principal focus is the point where are focused rays which were parallel before striking the surface. A plane at this point, perpendicular to the axis, is called the *second principal focal plane*. All rays parallel to a secondary axis will be focused in the second principal focal plane after being refracted at the surface.

4. Nodal Point, K. The nodal point is the center of curvature of the surface. All rays directed toward it pass through without being deflected. All straight lines passing from the object to the image, must, therefore, pass through it.

With the foregoing rules find the image-point of a luminous point on the axis.

In Figure 42 let L be the object-point, Φ the first principal focus, Φ′ the second principal focus, HH′ the refracting surface or principal plane, and K the nodal point. An image of the object-point, L, must be formed somewhere on the principal axis. If we can find the direction of another ray from L, the image-point of L will be located at the intersection of this line with the axis.

Take the ray LH′, which passes through the first principal focal plane at O and the principal plane at H′. If O be considered a luminous point the rays emitted will be parallel to one another after having passed through the surface, and, moreover, parallel to the direction of the ray OKP drawn through K. The ray OH′ may evidently be regarded as coming likewise from O. Hence it will be parallel to OP and directed toward Q. The ray H′Q meets the principal axis at L′ which is the image of L.

The same may be found by means of the next figure (43). AS, which passes through Φ, is parallel to the axis after having been refracted at the surface HH′S. This ray continues parallel to the principal axis until it crosses the secondary axis, AT, at A′. A′ is the image-point of A. The ray AH′ may also be used. This ray, being parallel to the axis, will be so refracted at the surface H′ that it will pass through Φ′.

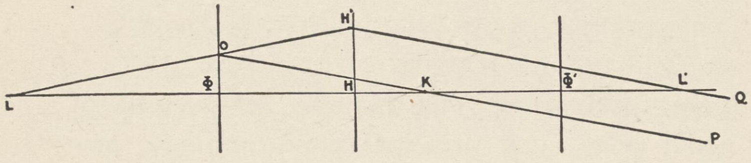


Figure 42.

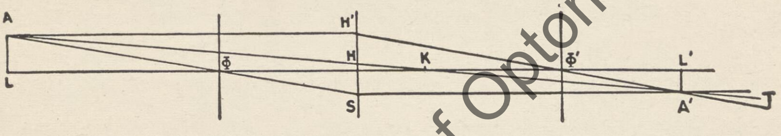


Figure 43.

Passage of Light through a System of Several Refracting Surfaces. There exist for every optical system, composed of whatever number of refracting surfaces whose centers are all situated on the same axis, three pairs of cardinal points, all situated on the axis, and two pairs of planes passed through four of these points perpendicular to the axis. Just as for a single refracting surface there are two principal focal points and planes; but instead of one nodal point, one principal point and one principal plane, there are two of each. The usefulness of the cardinal points and planes, according to the theory of Gauss, will be considered in the proper place.

CHAPTER VI.

LENSES.

A lens is a transparent body bounded by two surfaces, at least one of which is of such curvature that rays of light passing through will be made to converge or diverge. A lens may have a convex

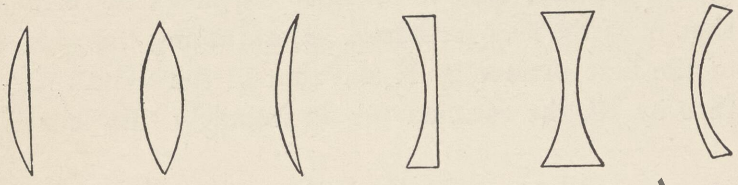


Figure 44.—Sections of lenses.

1, Planoconvex, 2, Biconvex. 3, Concavoconvex.
4, Planoconcave. 5, Biconcave. 6, Convexoconcave.

spherical, concave spherical, convex cylindrical or concave cylindrical surface, with the other surface plane; or the other surface may also have any one of these forms.

Lenses are planoconvex or planoconcave, that is, with one side plane and the other side convex or concave; biconvex or biconcave, when both sides are convex or concave; concavoconvex and convexoconcave (meniscus, periscopic), when one side is convex and the other is concave.

Infinitely Thin Lenses. For simplicity, we shall consider at present the lens to have negligible thickness and that the pencil of light is axial and thin. The thickness of the glass being disregarded, the surfaces, the principal points and the nodal points are all situated at the point where the principal axis passes through the lens. From this point all distances and lengths are measured.

The infinitely thin lens under consideration is surrounded by air, the index of refraction of which is 1, and we have only to consider the index of refraction, n, of the lens substance. Designating the radius of the first surface by R and that of the second surface by R′, the formula for an infinitely thin lens is

$$F = \frac{RR'}{(n - 1)\ (R + R')}$$

R taking the positive sign when convex, and the negative sign when concave. In a convex lens F is always positive; in a concave lens F is always negative; and in a meniscus lens F is either positive or negative, depending on whether the convex or concave side predominates.

In the planoconvex or planoconcave lens the radius of the plane surface is infinite and is not considered, so that the formula becomes

$$F = \frac{R}{n - 1}$$

which is the same as the formula for the principal focal distance in air for a single refracting surface. This is the only formula necessary to remember, because if the focal distance is found for each surface it is only necessary to add the reciprocals of these to find the power of the lens.

Refractive Power of Lenses. Diopters. The refractive power of a lens, which is expressed in diopters, is inversely proportional to the focal distance measured in meters.

$$D = \frac{1}{F}$$

or

$$F = \frac{1}{D}$$

where D expresses dioptric power. A lens having a focal distance of one meter has a power of

$$\frac{1}{1 \text{ meter}} = 1 \text{ diopter}$$

If the focal distance is two meters it will have a power of ½ or .50 D, and a lens of ½ meter focal distance will have a power of 2. D.

The formula for the planoconvex or planoconcave lens may, therefore, be written

$$D = \frac{n - 1}{R \text{ meters}}$$

and if D′ is the power of one surface and D″ the power of the other, the dioptric power of any infinitely thin lens is the sum of the powers of the two surfaces,

$$D = D' + D''$$

EXPERIMENT *to find the Principal Focus of an Infinitely Thin Convex Lens.* Set up a convex lens on the optical bench and face it toward a distant object. Adjust a screen behind the lens at the place where the clearest image is formed. The image is real and inverted. Measure the distance from the screen to the lens; remove the screen without disturbing the saddle-stand, and prove by parallax that the distance is correct. This is the direct method of finding the principal focal length of a convex lens.

Set up a plane mirror on one side of and facing the lens, and on the other side erect a perforated object-screen so that the rays, after being reflected by the mirror, pass through the lens a second time and are focused on the screen beside the luminous object.

It will be found that the screen is exactly in the principal focal plane (Figure 46) and the rays, being parallel after leaving the lens, are returned as parallel rays by the mirror, M, and consequently brought to a focus in the principal focal plane of the lens. The distance from the mirror to the lens does not affect the result.

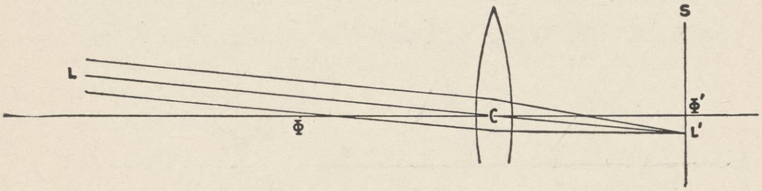


Figure 45.

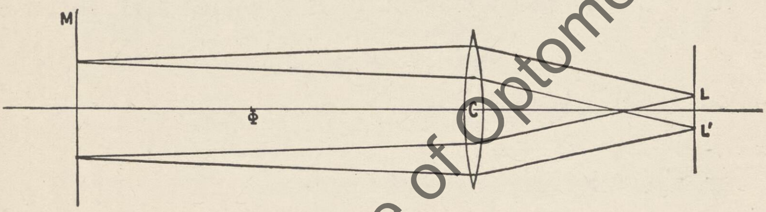


Figure 46.

Conjugate Foci of an Infinitely Thin Lens. Let L (Figure 47) be an object-point, L′ its image, O the point of incidence and C the center of a lens. If the aperture OC be considered a prism of small angle, the deviation of the ray may be considered constant. Let AO be a ray of light parallel to the axis of the lens. Make the angle AOΦ′ equal to

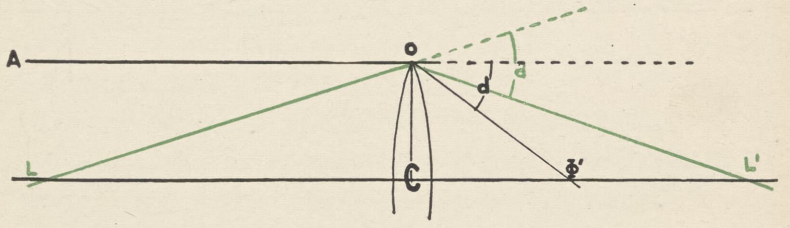


Figure 47.

LOL′; Φ′ will then be the principal focus for this lens. Since the deviation, d, is constant and, because

$$L'O\Phi' = AOL = OLC$$

$$d = OL'C + OLC = O\Phi'C$$

or, since the angles are very small,

$$\frac{OC}{CL'} + \frac{OC}{CL} = \frac{OC}{C\Phi'}$$

Dividing both sides of the equation by OC and calling $CL' = f'$, $CL = f$, and $C\Phi' = F$ it follows that

$$\frac{1}{f'} + \frac{1}{f} = \frac{1}{F}$$

For the concave lens we have (Figure 48)

$$d = OL'C - OLC = O\Phi'C$$

or

$$\frac{OC}{CL'} - \frac{OC}{CL} = \frac{OC}{C\Phi'}$$

and finally

$$\frac{1}{f'} - \frac{1}{f} = \frac{1}{F}$$

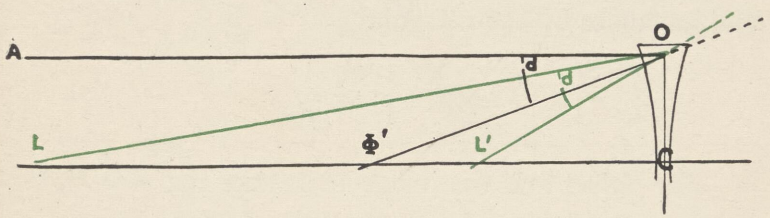


Figure 48.

in which F = the principal focal distance, f = the first conjugate focal distance and f′ = the second conjugate focal distance.

If the distances f and f′ are given in meters, and if D′ expresses the dioptric power of f, D″ the power of f′, and D the power of the lens, the formula may be written

$$D'' + D' = D$$

That is, the sum of the dioptric powers of the conjugate focal distances is equal to the dioptric power of the lens. The same formulæ were used in Reflection (page 56).

EXPERIMENT *to find the Conjugate Foci of an Infinitely Thin Convex Lens.* Set up a convex lens on the optical bench and place a luminous object at some distance beyond the principal focus. Carefully adjust a screen on the other side of the lens at the place where the clearest image is formed. Measure the conjugate focal distances, f and f′ (Figure 49), and calculate by the formula

$$\frac{1}{f'} + \frac{1}{f} = \frac{1}{F}$$

The object and the image both take the plus sign, being on opposite sides of the lens, and the image is real and inverted. The screen and the object may be interchanged with the same result. Erect pins instead of the screens and prove by parallax. Calculate by the formula $D' + D'' = D$.

EXPERIMENT. Set up a luminous object so that its distance from the lens is exactly twice the principal focal distance, as in Figure 50. Find the place where the image is formed. It will be found that the conjugate focal distances are equal ($f = f'$).

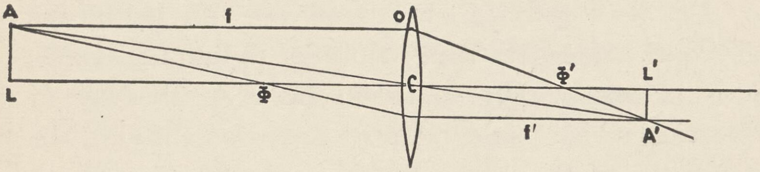


Figure 49.

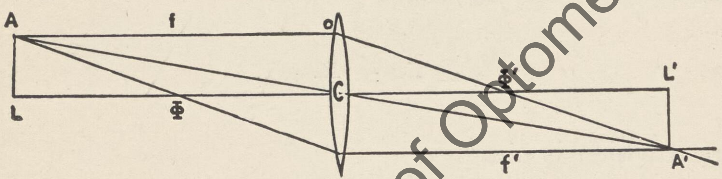


Figure 50.

EXPERIMENT. *The Relation of Size between the Object and Image formed by an Infinitely Thin Convex Lens.* Repeat the first experiment for finding the conjugate foci. Measure the distances f and f′, and the size of the object and of its image.

Let A (Figure 51) be a point not situated on the principal axis of the lens. In order to find the image, it is necessary only to draw, from A, the ray of direction which passes without deviation through the center, C, of the lens. Another ray, Ao, parallel to the axis, passes, after refraction, through the posterior principal focus, Φ′. The point A′, where these two rays intersect, is the image of A.

The perpendicular dropped to the axis, A′L′, is the image of AL. Signify the size of the object by O and the size of the image by I. From the similar triangles can be deduced

$$\frac{0}{\mathrm{I}} = \frac{\mathrm{f}}{\mathrm{f}'} = \frac{l}{\mathrm{F}} = \frac{\mathrm{F}'}{l'}$$

from which is obtained

$$\mathrm{I} = \frac{\mathrm{f}'0}{\mathrm{f}}.$$

$$\mathrm{I} = \frac{\mathrm{F}0}{l}$$

and

$$\mathrm{I} = \frac{l'0}{\mathrm{F}'}$$

As in a single refracting surface, the distances from the object and from the image are positive as long as they are on opposite sides of the lens, and the image is real and inverted.

Repeat the experiments for conjugate foci when the image is smaller than the object; when it is larger; when it is the same size. Verify the formulæ in each instance.

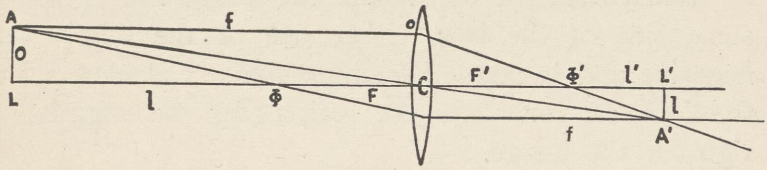


Figure 51.

EXPERIMENT. *The Virtual Image by an Infinitely Thin Convex Lens.* Place a pin or other object between the convex lens and its principal focus. On the same side of the lens erect another pin, as the image finder. Looking through the lens from the other side adjust the image finder by the method of parallax in the place where the image appears to be. The image is virtual and appears to diverge from where the rays would meet if projected backward, as in Figure 52.

When both the object and the image are on the same side of the lens, either one or the other is negative and it takes the minus sign. Calculate by the formulæ for conjugate foci, using the negative sign for the image.

EXPERIMENT *to find the Principal Focus of an Infinitely Thin Concave Lens.* Place a concave lens on the optical bench and face it toward a distant object. On the same side of the lens as the object, erect a pin as the image finder. Looking through the lens, adjust the image finder by parallax until it coincides with the position of the image.

The rays, after refraction, diverge as in Figure 53 and the rays from the image appear to diverge from the point where the refracted rays would meet if projected backward. The image is virtual and the posterior principal focus, Φ', lies in front of the lens. The same formulæ are applicable for the concave lens as for the convex except that F is negative in the concave lens.

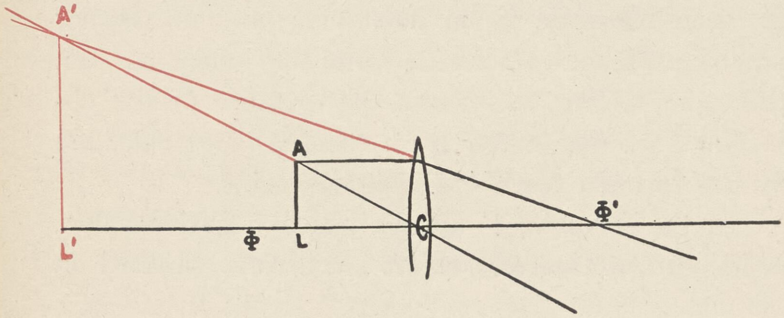


Figure 52.

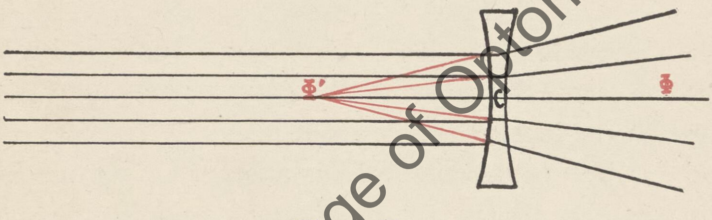


Figure 53.

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Northern Illinois College
4043 Drexel Bvd.

EXPERIMENT *to find the Conjugate Foci of an Infinitely Thin Concave Lens.* With the concave lens and an object-pin placed on the optical bench at some distance from the lens, adjust another pin by parallax to coincide with the image as seen through the lens. Measure the distance of the object and of the image from the lens and calculate by the formula for the conjugate foci.

In the formula, f′ always has a negative value, *i.e.,* the image and the object are always situated on

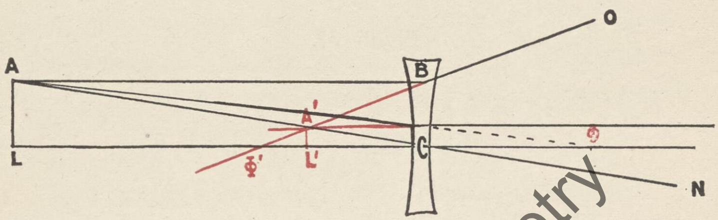


Figure 54.

the same side of the lens. Moreover f and f′ increase and decrease together as the image and object approach the lens or recede from it.

Let the negative focal distance, Φ′C, Figure 54, be equal to 10 cm. or 10 diopters, and let the distance of the object, f, equal 50 cm. or 2 diopters. By the formula D′ + D″ = D we say, since D is negative,

$$-10\ D = 2\ D' - 12\ D''$$

The distance of the virtual image from C (f′), having a dioptric power of —12, is 8.33 cm., and is on

the same side of the lens as the object. The image, by a concave lens, is always virtual and erect; the first conjugate focal distance is always greater than the second; and the image is always smaller than the object. The ray ACN passes through the lens without deviation, because it crosses through C. The ray AB, parallel to the axis, passes in a direction as if it came from the second focus, Φ, and is directed toward O.

The Combination of Infinitely Thin Spherical Lenses. If a lens is placed in apposition to another the strength of both is evidently combined. If the focal distance of the first lens is F′ and the focal distance of the second is F″, we have

$$\frac{1}{F'} + \frac{1}{F''} = \frac{1}{F}$$

or, if expressed in diopters,

$$D' + D'' = D$$

where F is the principal focal distance, and D the power, of the combination.

Combining a lens having a focal distance of +166 mm. with another of +250 mm. we obtain

$$\frac{1}{166} + \frac{1}{250} = \frac{1}{100}$$

or

$$6\ D + 4\ D = 10\ D$$

That is to say that both together act as a lens having a positive focal distance of 100 mm., or a power of +10 diopters. The same is true of three or more lenses. If a convex lens is added to a concave lens they partially or wholly neutralize each other and, instead of adding, the lesser is subtracted from the greater, and the sign of the greater predominates.

EXPERIMENT. Set up two lenses, close together, and prove the foregoing; first, with convex lenses,

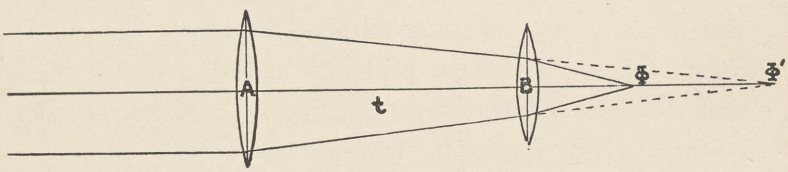


Figure 55.

and then with a convex combined with a concave lens. Separate the lenses, and take accurate measurements of the separation and the change in the focal power. When the lenses are not in contact their separation must be taken into account.

Take two lenses, A and B, Figure 55: A has a focal distance $A\Phi' = F'$; B has a focal distance F''. The parallel rays from the left would be refracted by A towards its focus, Φ'; and at a distance $AB = t$, behind the lens A, they strike the lens B. Up to this they still converged toward Φ', which is no longer separated from B by the distance F', but by $F' - t$. The convergence of the rays, when they

reach B, is expressed by $1/F' - t$, because they strike B as though from a lens with a focal distance of BΦ′. That is, the power of the rays, which are incident at B, is $1/B\Phi'$. This is fully explained in the discussion of light pencils (page 15). The combination of the two lenses gives

$$\frac{1}{F'-t} + \frac{1}{F''} = \frac{1}{Fe}$$

or

$$\frac{1}{B\Phi'} + \frac{1}{F''} = \frac{1}{Fe}$$

where Fe is the distance from B to the principal focus, Φ, of the system, and $1/Fe$ is the effective power measured from the second component of the combination.

Suppose F′ equals 250 mm., F″ equals 166 mm., and the separation is .50 mm., then

$$\frac{1}{250\text{-}50} + \frac{1}{166} = \frac{1}{200} + \frac{1}{166} = \frac{1}{90}$$

That is to say that the principal focus of the combination lies at a distance of 90 mm. behind B.

The same may be worked out in diopters in this way: the focal power of A is 4 diopters and its focal distance is 250 mm.; but as B is 50 mm. from A, and therefore 200 mm. from Φ′, B receives the rays as though they came from a lens with the focal power of $1/200$ or 5 diopters; and the power of B (6 D)

added to the power of the rays from A equals 11 diopters effective power at the second lens.

Combine a convex lens (A, Figure 56) having a focal distance of F′ with a concave lens, B, having a focal distance —F″, and separate them by a space t.

The parallel rays are refracted by the lens A toward Φ′, but on reaching B the convergence of the rays is $^1/F' - t$. The result is, letting $F' = 250$ mm., $F'' = -166$ mm. and $t = 50$ mm.,

$$\frac{1}{250\text{-}50} - \frac{1}{166} = -\frac{1}{976}$$

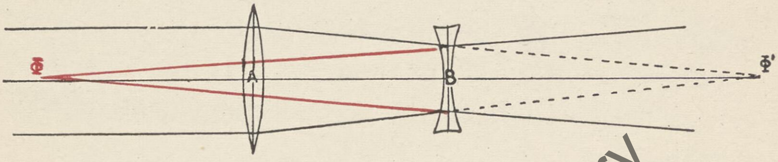


Figure 56.

That is, the combination is equivalent to a concave lens with a focal distance of 976 mm. from B, or a negative power of about 1. diopter at B. The rays, parallel before reaching the lens A, take such a direction that when striking B they have the power of being bent toward Φ′, and therefore a focal distance of BΦ′; on leaving B they take a direction as though they came from a point 976 mm. in front of B.

With a system of this kind the direction of the rays can, by shifting the position of the lenses, be varied at will. This is the principle of the Galilean telescope and of many optometers.

Combine a convex lens of $F' = 100$ mm. with a concave lens of $-F'' = 50$ mm. When these lenses are brought in contact we have

$$\frac{1}{100} - \frac{1}{50} = - \frac{1}{100}$$

or

$$10 \text{ D} - 20 \text{ D} = -10 \text{ D}$$

When they are separated by $t = 50$ mm. we have

$$\frac{1}{100\text{-}50} - \frac{1}{50} = \frac{1}{50} - \frac{1}{50} = 0$$

that is, they neutralize each other. When they are 80 mm. apart

$$\frac{1}{100\text{-}80} - \frac{1}{50} = \frac{1}{20} - \frac{1}{50} = + \frac{1}{33}$$

or

$$50 \text{ D} - 20 \text{ D} = 30 \text{ D}$$

When the rays come from the side of the concave lens the formula is necessarily changed.

Cylindrical Lenses. A cylindrical lens is one which, being the segment of a cylinder, has refractive power in all meridians except one; this one, being perfectly plane, is called the *axis of the lens.* The full strength of the lens is in the meridian of greatest curvature, at right angles to the axis, and this and the axis are called the *principal meridians.*

A *plano-cylinder* may be convex or concave. When a cylinder is combined with a spherical lens it

is called a *sphero-cylinder.* A sphero-cylinder may be a combination of a convex cylinder and a convex spherical lens; concave cylinder and concave spherical; or the combination may be of cylinder and sphere of opposite denomination. In the latter case it is called a *mixed cylinder.*

EXPERIMENT *to find the Focal Line of a Plane Cylinder.* Set up a convex plano-cylinder (about 5 D) on the optical bench and find the place where

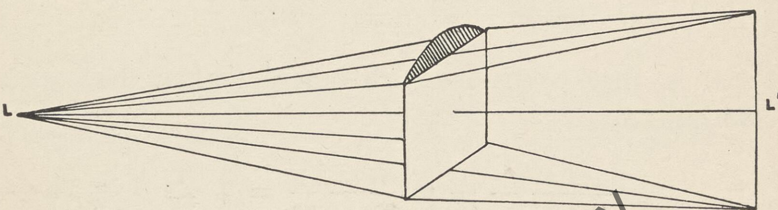


Figure 57.

the clearest image of a small circle of light is formed. This image will be found to be a line (L′, Figure 57).

A cylinder has, in the meridian of greatest curvature, the same action and deviating power as a spherical lens of the same curvature and index.

Parallel rays passing through a cylinder will meet at the principal focus, forming a line parallel to the axis of the lens. This line is called the *focal line* and lies at the principal focal distance (F) of the cylindrical lens.

EXPERIMENT. *Crossed Cylinders.* Combine two cylinders, each about 5 diopters, with their axes

parallel, the result is a cylindrical lens having the sum of the powers of the two cylinders. Turn them so that their axes are at right angles to each other, and it will be found that the image of a point is now a point. This combination is called a *crossed cylinder*. Crossed cylinders of equal strength and denomination are equivalent to a spherical lens of the same focal length.

Crossed cylinders of unequal strength form a system composed of a spherical portion, which is equivalent to the power of one of the component

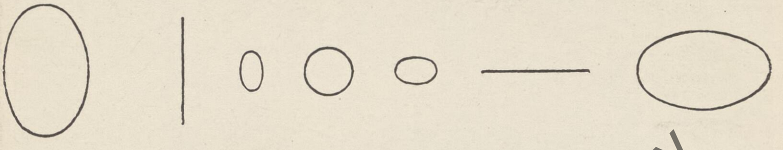


Figure 58.

cylinders, and a cylindrical portion equal to the difference between the powers of the component cylinders.

The intersection of the axial planes constitutes the principal axis of the lens system. Repeat the experiment with crossed cylinders of the same denomination, but of unequal strength—one of about 5 and the other 8 diopters. Two focal lines will be formed, one for each cylinder of the combination.

If the screen is first placed close to the lens and then gradually moved away, the rays will form the various diffusion images shown in Figure 58. First, a circle of light (not shown in the diagram),

then a broad oval which, gradually narrowing, forms the *first focal line* parallel to the axis of the strongest cylinder of the combination. As the screen is pushed farther away this line broadens to form a circle, then an oval in an opposite meridian, and finally it narrows down until it forms a *second focal line* at

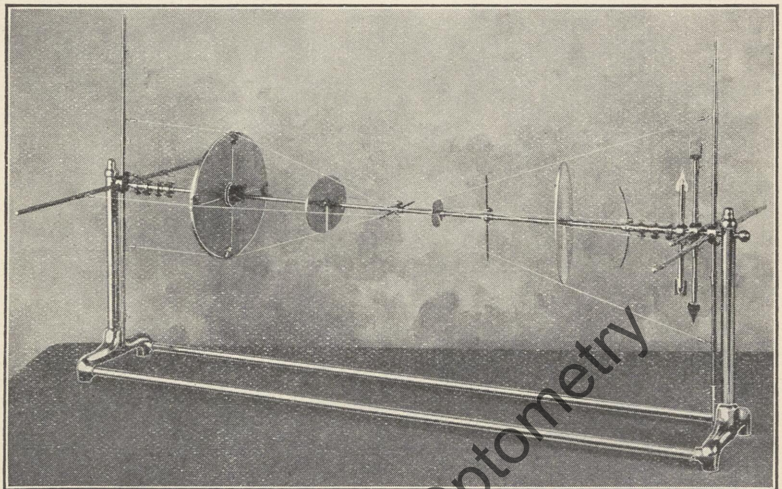


Figure 59.—Holloway's demonstration eye.

right angles to the first. This second focal line is parallel to the axis and is formed at the principal focal distance of the weakest cylinder of the combination. The distance separating the two lines is called the *interval of Stürm;* the whole system is known as *Stürm's Conoid.*

The Holloway Demonstration Eye is an excellent instrument for demonstrating almost any optical

system, but it is especially useful for illustrating the three dimensions of Stürm's Conoid. As is readily seen in the illustration, the horizontal bar is considered the principal axis of the system. Along this are various movable discs, cross-bars and sliding hooks to which may be attached the elastic cords that represent the light rays.

EXPERIMENT. *Cylinders with Axes at Oblique Angles.* Set up two cylinders with their axes at an oblique angle. It will be found that the first and second focal distances are different than when the axes were at right angles, and the axis of the system will lie in a different meridian than that of either axis of the component lenses. Cylinders with their axes at oblique angles are optically equivalent to a sphero-cylindrical lens.

The power of a cylinder varies from O along the axis to its full power in a meridian at right angles to the axis. A simple method for approximately finding the power in a certain meridian of any cylinder is to take the number of degrees of this meridian from the axis, multiply it by the power of the lens and divide the product by 90 —

$$\frac{\text{Number of degrees} \times \text{power of lens}}{90}$$

In a combination of obliquely crossed cylinders of equal strength and denomination, the axis of the resultant sphero-cylinder is midway between the two

axes of the component cylinders. When one cylinder is stronger than the other the axis will bear toward that of the stronger lens in the ratio of their powers, but the axis will always be somewhere between the axes of the components. When the cylinders are of different denomination the axis of the system will always lie outside of one or the other axis of the component lenses—depending on whether minus or plus sign is used for the cylindrical portion of the system—and at a distance from either axis depending on the ratio of the powers of the component lenses. The exact method for computing the resultant sphero-cylindrical lens and its axis in obliquely crossed cylinders has been described by Jackson, Prentice and others.

Toric Lenses. A toric lens is one with a surface having different radii of curvature, the least and greatest curvatures being at right angles to each other. Deep curved cylindrical lenses are possible only when one surface is toric.

Neutralization of Prisms and Lenses. It has been shown that if an object is viewed through a prism it will apparently be displaced toward the apex. At a distance of one meter a prism of one diopter (Δ) will displace an object one centimeter. The maximum power is in the meridian perpendicular to the base-apex lines, and the deviation decreases until, in the meridian parallel to these, it is zero.

Look at a line through a prism, turn the prism until a meridian is reached where the line passes through without a break and mark this with a glass pencil. Now turn the prism so this line is perpendicular to the object-line, and at a distance of one meter measure the displacement.

Dr. S. Lewis Zeigler has devised a convenient scale for the neutralization of prisms, which consists of a horizontal and vertical scale adjusted to

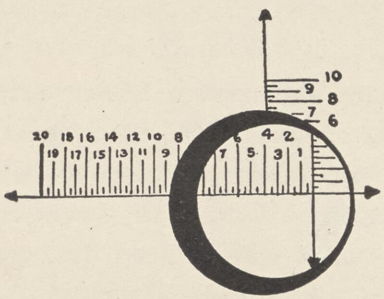


Figure 60.

measure the deviation in centimeters at a distance of one meter. The prism is held about 9 inches from the eye, exactly 1 meter from the scale and carefully turned so that the base, seen through the prism, is continuous with the base line of the scale. Figure 60 shows the scale seen when a prism displaces the vertical line 4 so that it coincides with the vertical base line. As the numbered lines are one centimeter apart, the prism is one of 4 Δ.

The rule given for the approximate power of cylinders in the meridians between those of maximum and minimum power may also be applied to prisms.

If two prisms be placed with their bases in apposition as in Figure 61 it will be seen that when looking through the point O, in a direction along the base line, there can be no displacement, since the action of one prism is neutralized by that of the other. If the prisms are moved so that the object is seen on either side of O the object will be apparently

Figure 61.

displaced toward the apex of either prism. In this case the displacement will be in a direction opposite to that in which the prisms are moved. When the prisms are arranged with their apices in apposition, as in Figure 62, it will be seen that the object apparently moves in the same direction as that of the prisms.

Figure 62.

The convex lens acts as two prisms with their bases towards each other, and a concave lens acts as two prisms with their apices towards each other. There is one place in either of these lenses where there is no displacement, and this is exactly at the summit of the curve.

The form of a cylinder is such that its power varies from zero along the axis to the full strength

in the meridian perpendicular to the axis. In a spherical lens the action is the same in every meridian because we may consider it to be composed of an infinite number of prisms, one in every meridian.

If an object is looked at through a convex lens, held at about fifteen inches from the eye, and moved from side to side or up and down, the object will always appear to move in an opposite direction to that of the lens. With a concave lens the apparent movement of the object is with the movement of the lens. There will be displacement of the object in every meridian except along the axis of a plano-cylinder, and, on turning the lens, a straight line will be bent with or against the direction of the lens movement, depending on whether the cylinder is concave or convex (Figure 63, f). This twisting also takes place with a sphero-cylinder. In the case of a spherical lens there is no apparent twisting of the object on turning the lens.

When two lenses of opposite denomination and of equal strength are placed together they are said to neutralize each other, and there will be no apparent movement of an object seen through the combination.*

Look at a straight line (edge of a window) through a lens; except through the center, the line

---

*This is only applicable to thin lenses of less than 9 diopters in the flat form and up to about 4 diopters in the deep curves. Plano-convex and plano-concave lenses, having the same curvature and index of refraction, are the only ones that exactly neutralize each other (see pp. 143-144).

will appear to be displaced towards the thinnest portion of the lens (Figure 63, a and b). Find the place where the line seems to pass through the lens without a break, and mark the edges of the glass here with a glass pencil (Figure 63, c). Turn the lens 90 degrees and mark this meridian in the same way. Draw lines through these points. The intersection of these two lines corresponds to the point which, in ophthalmic lenses, is called the *optical center*.

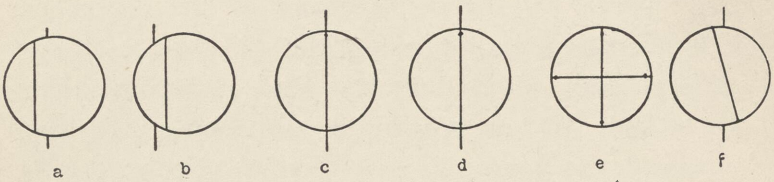


Figure 63.

In a spherical lens, so long as the line passes through the optical center, there will be no break in any meridian, but there will be apparent displacement in every direction of movement.

In a plano-cylinder there is no apparent displacement along the axis, and no twisting of a line along or perpendicular to the axis; but when accurately marked in the direction of the axis the line passes exactly through the axis of the lens without a break.

In a sphero-cylinder one line passes through the axis without a break, and another passes through the meridian perpendicular to this in the same way.

Mark the lens in both meridians. An object looked at through this lens will appear to move in one direction more than in the other and will appear to twist on turning the lens, just as in a plano-cylinder.

Now, with the proper lens, neutralize the weakest meridian. In the spherical lens all meridians have the same power; in the plano-cylinder the power is zero along the axis; in the sphero-cylinder the

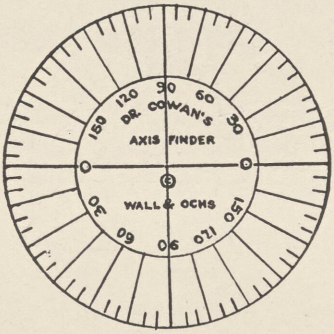


Figure 64.—Protractor.

weakest meridian is the spherical component and will be along the axis of the remaining cylinder, which should be neutralized in the opposite meridian.

Lay the marked lens on a protractor—made for this purpose—so that its geometric horizontal axis lies along the O—180 meridian, and so that its optical center coincides with the center of the circle. The direction of the axis of the lens can then be noted.

In order to obviate the necessity of marking the lens the author has devised a transparent pro-

tractor which can be applied directly to the lens, and the axis immediately noted by looking through the lens and the protractor at the same time.

Thick Lenses. In order to study the passage of light through a thick lens it is necessary to know the properties of the cardinal points and planes of an optical system.

According to the theory of Gauss, for a single refracting surface there are two principal focal points and planes, one principal point and plane and

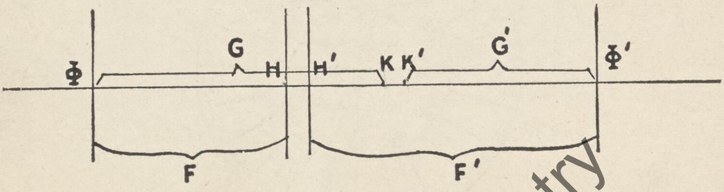


Figure 65.

The cardinal points and planes of an optical system according to the theory of Gauss. Φ, Φ', principal focal points and planes. *H*, *H'*, principal points and planes. *K*, *K'*, nodal points.

a center of curvature; but there exist for every optical system, composed of more than one spherical surface whose centers are all situated on the same axis, three pairs of cardinal points, all situated on the axis, and two pairs of planes passed through four of these points perpendicular to the axis (Figure 65).

Nodal Points, K. K'. The nodal points are so situated that every ray which before being refracted

is directed toward the first, seems after refraction to come from the second, and takes a final direction parallel to that which it had at first. These two parallel rays are called *lines of direction* and act, in the combined system, the same part as the line passing through the nodal point or center of curvature of a single refracting surface. K and K′ are images of each other.

Principal Points and *Principal Planes, H, H′.* When the incident ray, prolonged if necessary, passes through a point on H the corresponding emergent ray or its prolongation passes through H′, but the incident ray is not parallel to the emergent. If a parallel to the principal axis is drawn through the point at which the incident ray pierces the first principal plane, the point where this line pierces the second principal plane is in the course of the corresponding emergent ray or its prolongation. The direction of any incident ray and of the corresponding emergent ray pierces the first and second principal planes in two points situated on the same side of and at the same distance from the principal axis of the system. The second principal plane is an optical image of the first and *vice versa.* These are the only two conjugate images which have the same size and direction. The two principal planes of a compound system correspond to the single principal plane of one refracting surface.

Principal Foci and *Principal Focal Planes, Φ, Φ′.* The principal foci and planes have the same property

as in a single refracting surface. Rays which were parallel before refraction are focused in a point in the principal focal plane lying on the opposite side of the system.

The *first principal focal distance,* F, is the interval which separates the first principal point from the first principal focus. The *second principal focal distance,* F′, is the interval which separates the second principal point from the second principal focus.

It will be seen in Figure 65 that

$$\Phi K = \Phi' H' = G = F'$$

and

$$\Phi H = \Phi' K' = F = G'$$

From this it follows that the respective distance of each H and K of the same kind is equal to the difference of the two focal distances

$$HK = H'K' = F' - F$$

and

$$HH' = KK'$$

Finally, the two principal focal distances are to each other as the indices of refraction of the first and last media are to each other.

$$\frac{F}{n} = \frac{F'}{n''}$$

or

$$\frac{F}{F'} = \frac{n}{n''}$$

if n″ be the index of refraction of the last medium. If the last medium is the same as the first and we have n = n″ (as in most optical instruments, but not in the eye) the two principal focal distances are equal and the principal points coincide with the nodal points.

The theory of Gauss was developed in 1841, and is a comparatively simple method of determining the procedure of light through a number of refracting surfaces separating media of the same or different indices of refraction.

With a knowledge of the principles laid down, and provided the position of the cardinal points are known, it is a simple matter to trace the rays through an optical system and to find the size and position of the image of an object. The theory of Gauss, however, applies only to a centered optical system and for rays very close to the principal axis. These so-called "paraxial rays" lie within what is known as the Gaussian space.

It will be found that the same formulæ which were used for a single refracting surface and for infinitely thin lenses, may be used for finding the cardinal points and the conjugate foci of a thick lens or any other optical instrument.

Optical Center of a Biconvex Lens. From J on the surface S (Figure 66) draw a line to the center of curvature, C′, of this surface. From the center of curvature, C″, of S′ draw C″ J′, parallel to C′J. The planes tangent to the refractive surfaces at J and J′ are parallel, since they are perpendicular to the parallel lines C′J and C″J′. Hence if the ray TJ

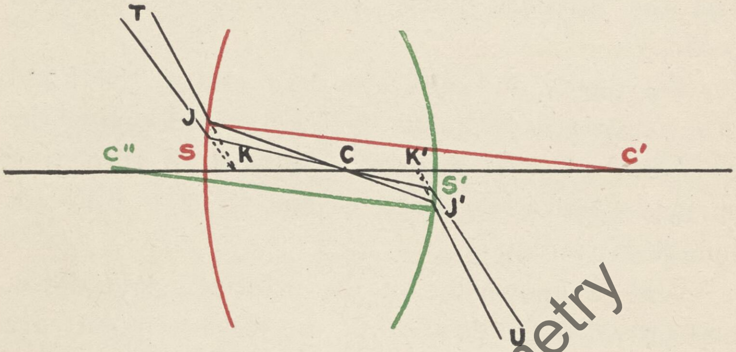


Figure 66.

meets the first surface at such an angle that it follows JJ′ in entering, the corresponding emerging ray, J′U, will be parallel to the incident ray, for the ray will thus have passed through a refractive medium limited by parallel surfaces.

Connecting JJ′ we find, where this line cuts the axis, the point C which is called the *optical center*. The position of the point C results from the similarity of the triangles C′JS and C″J′S′, and of the triangles CSJ and CS′J′—

$$\frac{C'S}{C''S'} = \frac{SJ}{S'J'} \qquad \frac{SJ}{S'J'} = \frac{CS}{CS'} \qquad \frac{CS}{CS'} = \frac{R}{R'}$$

where R and R′ are the radii of curvature of the first and second surfaces respectively.

This is equal to saying that in order to find the point C the thickness of the lens must be divided into two parts which will be to each other as the radii of the corresponding surfaces. Hence C is midway between the two surfaces of the lens when they are of equal curvature, and nearer the more convex surface when they differ.

Every incident ray, refracted by the first surface in such a way as to pass through the optical center, emerges from the system in a direction parallel to its primitive one.

Nodal Points of a Biconvex Lens. Consider C (Figure 66) an object-point, and CJ a luminous ray directed toward the first surface, S. The ray will be refracted along JT and a virtual image of C will be found where JT, prolonged backward, cuts the axis at K. This is called the *first nodal point*. The *second nodal point*, K′, found in the same way, is the virtual image of C by the second surface, S′. The nodal points are images of C and of each other, so that a ray directed toward one appears to come from the other, and the emergent ray is parallel to the incident ray.

Principal Points and Planes of a Biconvex Lens. In order to ascertain the location of the two principal planes it may be assumed that both of them are images of a common luminous plane, seen from either side of the refractive system.

In Figure 67 a ray QN, parallel to the axis, will be refracted by the first surface, S, to its posterior principal focus, SΦ′. Another parallel ray from the opposite side, Q′N′, at the same distance from the axis as QN, will be refracted by the second surface,

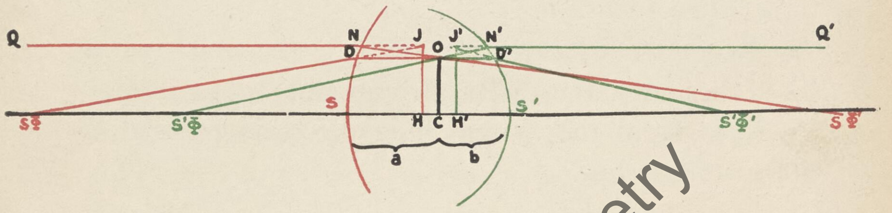


Figure 67.

S′, to its anterior principal focus at S′Φ. These rays will cross in a point, O, which lies in a plane perpendicular to the axis at the optical center.

Consider O a luminous point, and ON a ray directed toward S as though it came from its principal focus, SΦ′. The emergent ray, NQ, will be parallel to the axis. Another ray, OD, parallel to the axis, after refraction at S will be directed to the anterior principal focus, SΦ. The emergent rays, NQ and D SΦ, will form a virtual image at J where they will meet after being prolonged backward.

This is the image of O by the first surface. In the same way a virtual image of O by the second surface, S′, will be found at J′.

The point O may lie anyhere in a plane perpendicular to the axis through the optical center, and if O has been well chosen JH and J′H′ are images situated in the principal planes and must be of equal size.

It has been shown that

$$\frac{SC}{S'C} = \frac{R}{R'}$$

but there is another relation. To find the location of C we make use of the similar triangles SΦ′ S N and SΦ′ C O, whence we derive, designating C S by a and the posterior focal distance of the first surface by 1f′.

$$\frac{SN}{CO} = \frac{S\,S\Phi'}{C\,S\Phi'} = \frac{1f'}{1f'-a}$$

From the similarity of the triangles S′Φ S′ N′ and S′Φ C O, if we designate S′C by b and the anterior focal distance of the second surface by 2f, we have

$$\frac{S'N'}{CO} = \frac{S'\Phi\,S'}{S'\Phi\,C} = \frac{2f}{2f-b}$$

S N and S′ N′ being equal to J H which is equal to J′ H′, we have, according to our previous supposition

$$\frac{1f'}{1f' - a} = \frac{2f}{2f - b}$$

whence

$$\frac{a}{b} = \frac{1f'}{2f}$$

If we represent the anterior focal distance of the first surface by 1f and the posterior focal distance of the second surface by 2f′ we have

$$\frac{a}{b} = \frac{1f}{2f'}$$

That is to say, that the distance of the optical center from the first surface is to its distance from the second surface, as the principal focal distance of the first surface in air is to the principal focal distance of the second surface in air; and, since the dioptric power is the reciprocal of the focal distance in air we may say that

$$\frac{a}{b} = \frac{D''}{D'}$$

where D″ represents the dioptric power of the second surface and D′ the power in diopters for the first surface.

In order to find C we merely divide the thickness, t, into the number of parts equal to the sum of the two radii and mark off the proper number of points from either surface. This is the same as saying

$$a = \frac{R}{R + R'} \times t$$

and

$$b = \frac{R'}{R + R'} \times t$$

in which the focal distances, either in air or in the glass, may be substituted for the radii of curvature of the respective surfaces.

If the dioptric powers of the surfaces are given, the formulæ are necessarily

$$a = \frac{D''}{D' + D''} \times t$$

and

$$b = \frac{D'}{D' + D''} \times t$$

The distance is measured from one surface toward the other when the result is positive, and in the opposite direction when negative.

Having found O, which is considered an object-point, J and J′ can easily be found for each surface by using the formula

$$ll' = FF'$$

Example. Consider a lens, 3.6 millimeters in thickness, with a power of 5 D for its first surface, 4 D for its second surface, and 1.5 for the index of refraction of the glass.

The point C would lie on the axis $\frac{4}{9} \times 3.6$ mm. $= 1.6$ mm. from the 5 D surface, and $\frac{5}{9} \times 3.6$ mm. $= 2$ mm. from the 4 D surface. The point O lies anywhere on a plane perpendicular to the axis at C. With O lying at a definite distance from the principal planes of S and S′ we may proceed to seek where an image is formed by each of these surfaces.

The focal distance of S in air is 200 mm. and, since the index of refraction is 1.5, its focal distance in the glass is $1.5 \times 200 = 300$ mm. Therefore

$$F \times F' = 200 \times 300 = 60000$$

$$l = 300 - 1.6 = 298.4$$

and l takes the negative sign because it lies to the surface side of the posterior principal focus. Therefore

$$ll' = -298.4 \times -201.07$$

Since the anterior principal focus lies 200 mm. in front of S, and l' is found to be -201.07, the image, J, must be virtual and lie at a point 1.07 mm. behind S.

For J′ we proceed in the same way, but in the opposite direction, towards the second surface. Here

$$FF' = 250 \times 375 = 93750$$

The object-point is 2. mm. from S′, so that

$$l = 375 - 2 = -373$$

and

$$ll' = -373 \times -251.34$$

The sign being negative, J′ is virtual and lies 1.34 mm. from S′ within the lens.

In a lens surrounded by a single medium the first nodal point coincides with the first principal point, and the second nodal point coincides with the second principal point.

Principal Focal Distances of a Biconvex Lens. The second principal focal distance of the lens (Figure 68) is the point Φ′ where are focused all the

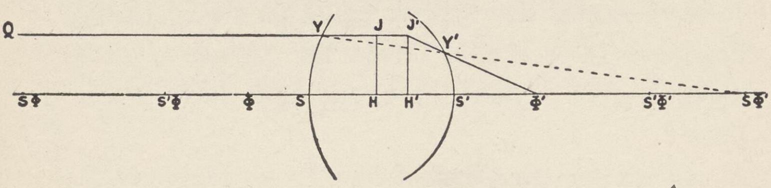


Figure 68.

rays which, before entering the lens, were parallel to the axis. Let QY be one of these rays. The surface S alone would refract it toward its second focus SΦ′, hence YY′ is the path taken by the ray in the interior of the lens. Y′ is a point through which the ray travels after refraction.

When a ray meets the first principal plane at J it continues its course after refraction as if it came from J′, on the second principal plane—JJ′ being parallel to the axis. Hence J′Y′Φ′ is the final direction of the incident ray QY, and Φ′ is the second principal focus of the lens.

The first principal focus is geometrically determined by the same construction, but with the rays in the opposite direction.

The principal focal distances, always measured from the principal planes, of a lens placed in a single medium are equal regardless of the radii of curvature of the surfaces; but the principal plane of the more convex surface being nearer to its surface than the principal plane of the less convex surface is to the latter, it is readily seen that, while the principal focal distances are always equal as stated above, the distances of Φ and Φ' from the surfaces are never equal except when these surfaces have the same power.

When we imagined a lens to be infinitely thin it was seen that

$$D = D' + D''$$

in which the dioptric powers of the surfaces were simply added. The same formula was used to find the power of two infinitely thin lenses placed in contact. This is usually done in practice when the thickness of the lens or lenses is negligible, but in a thick lens, just as in a combination of lenses, the distance between the two surfaces must be taken into consideration.

The difference between the focal power of a thick lens and that of an infinitely thin lens is the same, in principle, as the difference between the power of a

combination of two lenses when in contact and when there is a separation.

Suppose each surface to be an infinitely thin lens, separated by the thickness of the glass. Call the focal distance of the first surface in air F′ and that of the second surface in air F″. We may then write

$$\frac{1}{F' - t} + \frac{1}{F''} = \frac{1}{Fe}$$

where Fe is the distance from the second unit to the posterior focus of the combination. This formula is the same as that for two infinitely thin lenses in air (page 103), and it can be used for any two refracting systems where t is the distance between the two adjacent principal planes of the component units; but it is first necessary to reduce all distances to their values in the medium where Fe will be found.

In considering a thick lens the distance between the two surfaces on the principal axis is the distance between the principal points of each component, because for a single surface there is only one principal point and it lies at the intersection of the surface with the axis. The principal points, in the lens, are separated by glass instead of air, so that the index of refraction of the glass must be taken into consideration. The *reduced distance*, t, which signifies the distance between the principal points of the components of a system in air, now equals

$$\frac{d}{n}$$

in which d is the distance between the two surfaces, and n is the index of refraction of the glass.

Take the lens in the last example in which the thickness is 3.6 mm., the index of refraction is 1.5, first surface is 5 D and the second surface is 4 D in power. Here

$$t = \frac{3.6}{1.5} = 2.4 \text{ mm.}$$

That is to say that the thickness of the lens is equivalent to a distance of 2.4 mm. in air.

If the 5 D side of the lens is called the front surface and parallel rays be incident on this side we may write, since the focal distance of the front surface is 200 mm. and that of the back surface is 250 mm.,

$$\frac{1}{200 - 2.4} + \frac{1}{250} = \frac{1}{110.36}$$

The principal focus of this lens, measured from the back surface, lies at a distance of 110.36 mm. But the principal focal distance is measured from the principal point and this, for the side of the lens being considered, was found to be 1.34 mm. within the lens, so that the principal focal distance of our lens is 110.36 + 1.34 = 111.7 mm., a power of 8.952 D.

If the light is incident at the back surface (4 D side) the equation must be written

$$\frac{1}{250 - 2.4} + \frac{1}{200} = \frac{1}{110.63}$$

In this case the principal focus in front lies at a distance of 110.63 mm. from the first surface. The first principal point is within the lens, 1.07 mm. from the first surface, and the principal focal distance, being the same on either side, is 110.63 + 1.07 = 111.7 mm. or 8.952 D.

Gullstrand's formula for the dioptric power of two refracting units is

$$D = D' + D'' - t\, D'D''$$

where D is the dioptric power of the combined system, D′ the power of the first refracting unit, D″ that of the second and t the distance between the two adjacent principal points of the component units in air. Substituting the values of our lens in this formula, giving the reduced thickness, t, in meters,

$$D = 5 + 4 - .0024 \times 5 \times 4 = 8.952$$

When another lens is added the combined power of the first system is considered a unit and the added lens another unit. In this way the power of any number of component systems may be calculated.

Conjugate Foci of a Biconvex Lens. Having found the principal points and principal foci of a lens it is easy to find the conjugate foci, and also the relation between the sizes of the object and image formed by a lens.

Let H and H′ be the principal points of the lens (Figure 69), Φ and Φ′ the principal foci, and L an object-point situated on the axis. Draw a ray

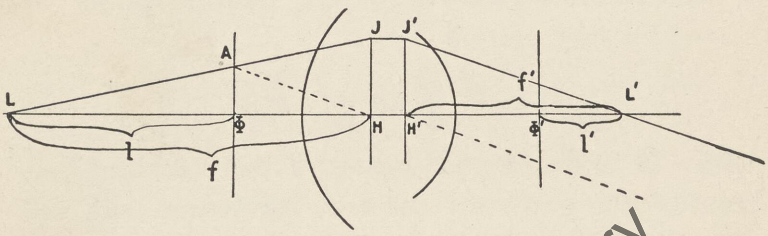


Figure 69.

LJ which cuts the focal plane at A and the first principal plane at J. Draw JJ′ parallel to the axis. J′ is the first point of direction for the emergent ray. Suppose A to be a luminous point. One of the rays from A may be directed to the first principal or nodal point and it will pass out of the system, appearing to come from H′, but parallel to AH. The point A, being situated in the focal plane, all rays emanating from it should, after refraction, be parallel to each other and to AH. AJ being considered as one of these rays, J′L′, parallel to AH, is the final direc-

tion of the ray, and L′ on the axis is the image of L.

The formulæ

$$\frac{1}{f} + \frac{1}{f'} = \frac{1}{F}$$

$$D' + D'' = D$$

$$ll' = FF'$$

may be used in the thick as well as the thin lenses except, of course, that the distances are measured from the principal points.

Example. In the convex lens of 111.7 mm. focal distance or 8.952 D power, the first principal point was found to be 1.07 mm. from the front surface, and the second principal point was 1·34 mm. from the back surface. They were both within the lens. If an object is placed at a distance of 1000 mm. in front of the first principal point we should find

$$\frac{1}{1000} + \frac{1}{125.75} = \frac{1}{111.7}$$

or

$$1.\ D' + 7.952\ D'' = 8.952\ D$$

that is, $f' = 125.7$ mm. and $D'' = 7.952$ diopters. This means that the image is formed at a distance of 125.75 mm. behind the posterior principal plane and, therefore, 124.41 mm. behind the back surface of the lens.

Size of the Image formed by a Biconvex Lens. Let AL (Figure 70) be an object situated in a plane perpendicular to the axis. Its image will be likewise situated in a plane perpendicular to the axis. In order to find the point where the rays from A are focused draw the line of direction AH. The image should be situated on the line H′A′, parallel to AH.

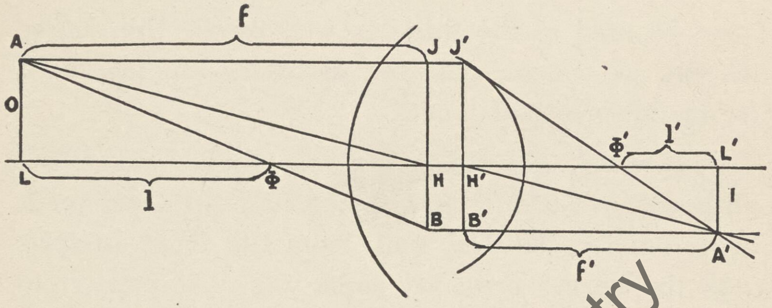


Figure 70.

Another ray given off from A, parallel to the axis, pierces the first principal plane at J, the second at J′ and emerges following J′Φ′A′. The point A′, in which these two rays intersect, is the image of A. Erect the perpendicular, and A′L′ is the image of AL. Another ray coming from A, passing through the first principal focus, will meet the first principal plane at B, pass on to the second at B′ and out of the lens parallel to the axis through A′.

As to the size of the image, on making

$$\begin{aligned} AL &= 0 \\ A'L' &= I \\ LH &= f \\ L'H' &= f' \\ L\Phi &= l \\ L'\Phi' &= l' \\ H\Phi &= F \\ H'\Phi' &= F' \end{aligned}$$

it can readily be seen that

$$\frac{0}{I} = \frac{f}{f'} = \frac{l}{F} = \frac{F'}{l'}$$

from which is obtained

$$ll' = FF'$$

$$I = \frac{f'0}{f}$$

$$I = \frac{F0}{l}$$

and

$$I = \frac{l'0}{F'}$$

as for the infinitely thin lenses.

Biconcave Lens. The cardinal points of a biconcave lens are determined in the same way as are those of the biconvex lens. The foci of the first surface as well as those of the second are virtual, and therefore all the focal distances are negative.

The optical center is again found by drawing two parallel radii to the surfaces and connecting these points. Where this line crosses the axis is the optical center and lies within the lens.

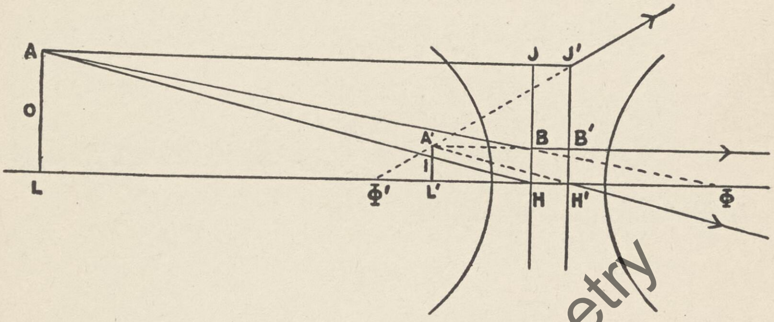


Figure 71.

The formulæ and constructions for the biconcave lens are the same as those for the biconvex. Since the focus is always on the same side as the real object (Figure 71), and therefore virtual, it must be considered posterior and measured from the posterior or second principal plane.

Planoconvex and Planoconcave Lenses. The optical center, first principal point and first nodal point coincide at the summit of the curved surface. The second principal point is situated in the interior of the lens.

Menisci. To find the optical center of a meniscus lens draw the parallel radii C′S and C″S′ (Figures 72 and 73). A line (JJ′) drawn through the points

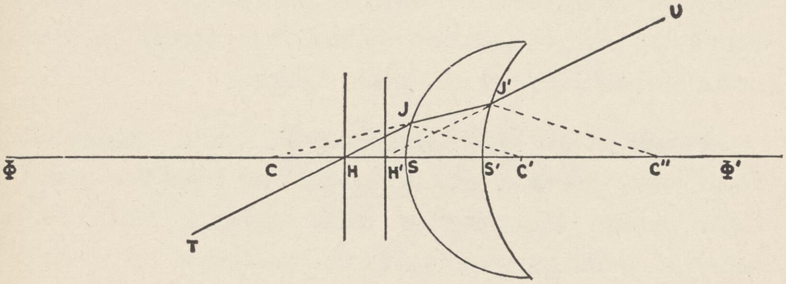


Figure 72.

where these radii intersect the surfaces will cut the axis in the optical center. Considering the ray JJ′ as if passing through the optical center, C, the in-

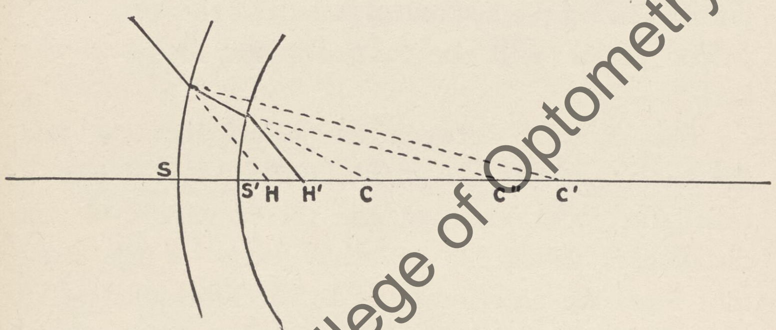


Figure 73.

cident ray TJ and the emergent ray J′U are parallel. The incident ray, prolonged if necessary as in the convexoconcave, cuts the axis in the first principal or nodal point. The emergent ray, prolonged if

necessary, cuts the axis in the second principal or nodal point. It results that the optical center and the principal points lie in front of the lens in the concavoconvex, and behind the lens in the convexo-concave. All values for menisci are found by the same formulæ as for the other shapes.

Power of an Ophthalmic Lens. The principal focal distances of a lens are measured from the principal points, the anterior focal distance from the anterior principal point and the posterior focal distance from the posterior principal point. It has been shown that, for a lens in a single medium, the anterior principal focal distance is equal to the posterior principal focal distance. The reciprocal of either the anterior or posterior principal focal distance is called the *equivalent power* of the lens. This power varies with the thickness and shape of the lens.

The distance between the principal focus and the surface of the lens on the corresponding side is called the *focal intercept*, and the reciprocal of this distance is called the *effective power* of the lens; that from the anterior focus to the first surface being the *front focal distance,* and from the second focus to the second surface being the *back focal distance* (see pages 128-131).

The power of a lens, measured from the surfaces, was called Scheitel refraction by von Rohr and is known as *vertex refraction.* Vertex refraction, there-

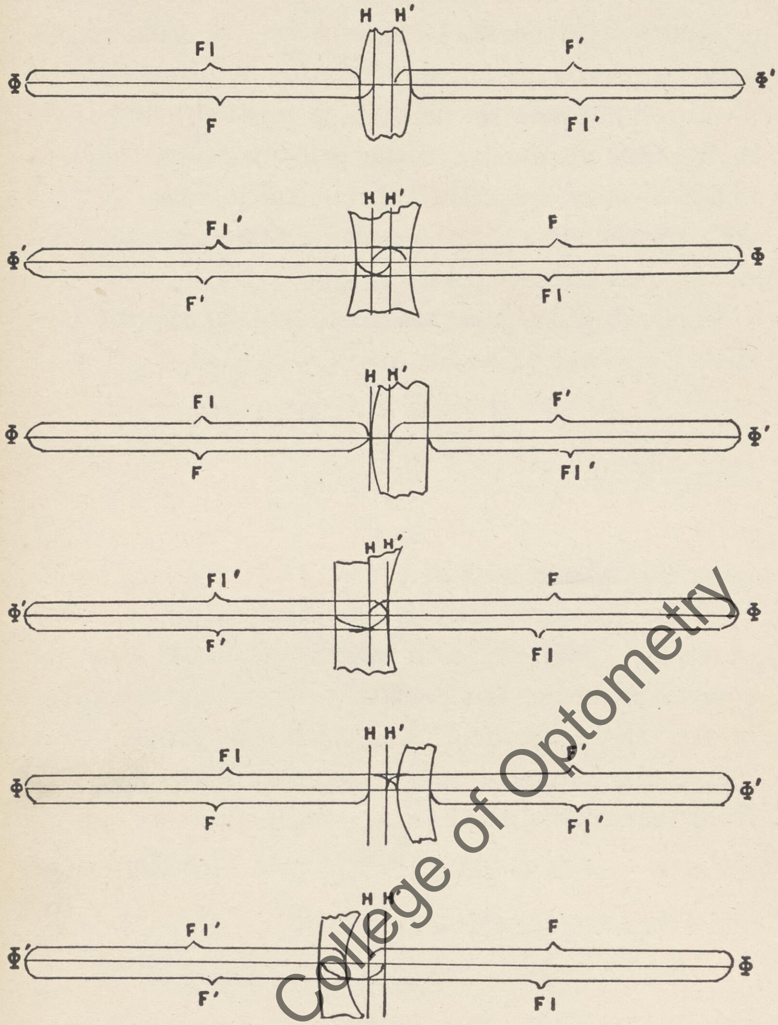


Figure 74.

FI represents the focal intercept and *F* the principal focal distance.

fore, is the effective power of a lens from the vertex or intersection of the surface with the axis of the lens.

If we suppose the lens to be infinitely thin there is no difference between the principal focal distance and the focal intercept, but in thick lenses or the deep menisci this difference is at times an important factor (see Figure 74).

The formulæ generally used for the approximate effective power of a lens are

$$D' + D'' + t\ D''^2$$

from the first surface, and

$$D' + D'' + t\ D'^2$$

from the second surface.

Take the same lens as was used in our previous examples (page 125) in which we found that the equivalent power was 8.952 D. Replacing the values in the above formula, and with the proper value for t, the effective power of this lens, measured from the front surface, is

$$5\ D + 4\ D + .0036/1.5 \times 4 \times 4 = 9.0384\ D$$

and from the second surface

$$5\ D + 4\ D + .0036/1.5 \times 5 \times 5 = 9.06\ D$$

These differences are so slight that they are negligible.

In the planoconvex and planoconcave lenses H lies on the curved surface, and H′ about ⅓ the thick-

ness within the lens. The back focus of the planoconvex lens is closer to S′ than the front focus is to S; the same is true of the planoconcave (Figure 74). In this case, as well as in the ordinary ophthalmic biconvex and biconcave lenses, the difference is extremely small. In the higher powers of the deep curved lenses the difference between equivalent and effective power cannot be neglected, and this difference must be taken into consideration when ordering lenses of various shapes.

Take a lens of the same thickness and index of refraction as in the last example, but $D' = +15$ D and $D'' = -6$ D, and find the equivalent power. Here

$$D' + D'' = +15 - 6 = +9 \text{ D}$$

and

$$t/nD'D'' = .0036/1.5 \times -90 = -.2160$$

so that

$$+9 \text{ D} - -.2160 = +9.2160 \text{ D}$$

By the formula for the vertex refraction at the first surface of this lens, we have

$$+15 \text{ D} - 6 \text{ D} + .0036/1.5 \times (-6 \times 6) = +8.9136 \text{ D}$$

and for the back focal power

$$+15 \text{ D} - 6 \text{ D} + .0036/1.5 \times (+15 \times 15) = 9.54 \text{ D}$$

a difference of more than .62 D between the back focal power and the front focal power.

The same result will be found by the formula

$$\frac{1}{F' - t} + \frac{1}{F''} = \frac{1}{Fe}$$

as used for two infinitely thin lenses (page 103) or for a thick lens (page 129).

Suppose this lens, 3.6 mm. in thickness, plus 15.00 D curve in front and index 1.5 is desired to have exactly 9.00 D back focal power. The focal distance of the front surface is 66.666 mm. and the reduced thickness is 2.4 mm. We write

$$\frac{1}{66.666 - 2.4} = \frac{1}{64.266}$$

That is, the focal distance of the front surface is 64.266 mm., and its dioptric power

$$D = \frac{1000}{64.266} = 15.56$$

at the vertex of the back surface. In order that the back focal power of the lens be exactly 9.00 D we must add

$$+ 9.00 - 15.56 = - 6.56 \text{ D}$$

Therefore a lens 3.6 mm. in thickness, 1.5 index, plus 15.00 D front surface and —6.56 D back surface has a back focal power of 9.00 D.

EXPERIMENT. Set up a deep curved concavo-convex lens on the optical bench and carefully meas-

ure the focal intercepts. Since the principal points lie close together in the ordinary ophthalmic lens they may be considered one point. The sum of the two distances, allowing for the thickness of the lens, and dividing by 2, will give the approximate location of these points.

It is necessary to consider the effective power in

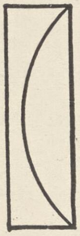


Figure 75.

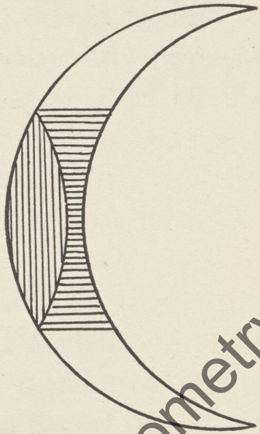


Figure 76.

the neutralization of lenses. Prentice showed that the only lenses that accurately neutralize each other are the planoconvex with planoconcave. In this case the first principal point of one coincides with the second principal point of the other (Figure 75) and the two together form a plane plate.

Figure 76 illustrates the reason why two equally curved lenses of opposite denomination cannot neutralize each other. It will be seen that if their surface lines are extended the figure becomes the section

of a meniscus convex lens. However, for the lower powers, the ordinary method of neutralization of lenses as described on pages 110-116 is accurate enough for all practical purposes.

Aberrations of Lenses. The most important aberrations are spherical aberration, chromatic aberration, curvature of field, distortion, and astigmatism of oblique pencils.

Spherical Aberration. It was seen in Figure 33 that all the rays from a luminous point cannot be united in a single point after refraction by a spherical surface, but only the rays that traverse the same circle with the axis as center are united in the same point. The result is a series of points along the axis, the closest of which is formed by the marginal rays and the farthest by the rays lying very close to the axis, forming a focal line. The distance between the focal length of the marginal rays and that of the axial rays (L′L″, Figure 77) constitutes the longitudinal aberration.

It follows that a large lens (aperture) will give greater spherical aberration. If the aperture is small enough the aberration is practically eliminated.

In Figure 77 the image at L′ is that formed by the axial rays and therefore the sharpest. It is surrounded by a halo (AB) formed by the diverging marginal rays which have already crossed at L″. At L″ there will be a sharp central image by the marginal rays, surrounded by a halo of converging

axial rays on the way to their focus at L′. We call this *positive* aberration.

In the case of concave lenses or in aspherical surfaces where the periphery has a longer focal distance than the center, it is called *negative* aberration.

There is no aberration when the ratio of the sine of the angle of incidence to the sine of the angle of refraction is the same for both the axial and the

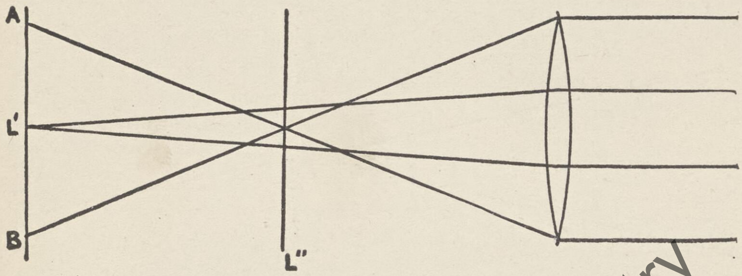


Figure 77.

marginal rays. This is the ideal for any optical system and is called the *sine condition.*

Chromatic Aberration. We have supposed a spherical lens to be composed of an infinite number of prisms. Since prisms disperse white light, lenses actually have a different focus for each color; the difference increasing towards the periphery. The refrangibility of the violet light being greatest its focal distance is shortest, while the least refrangible, red, has the longest focal distance. The focal distances of the other colors, indigo, blue, green, yellow and orange, vary between the violet and the red.

Chromatic aberration is greater in proportion as the lens is stronger and as the incident rays approach the margin.

In both spherical and chromatic aberration it must be remembered that as the focal distances vary so does the magnification of the image vary according to the formulæ for the relative sizes of object and image.

Achromatic lenses are made by cementing a

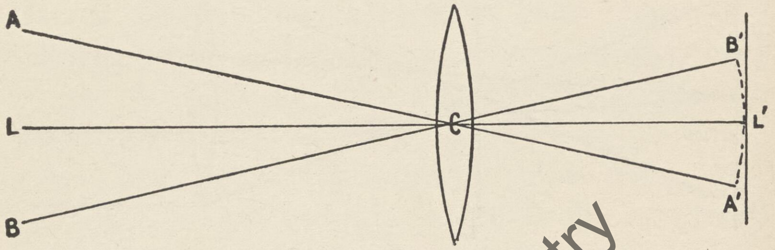


Figure 78.

crown glass to one of flint with proper curvature. Chromatic aberration must be corrected in most optical instruments, but not in spectacle lenses where the chromatic defects can usually be disregarded.

Curvature of Image. The image of a flat object cannot be received on a flat screen. The rays AC, LC and BC (Figure 78) incident from an object at such a distance that their lengths may be considered equal, will emerge along CB′, CL′ and CA′ and an image will be formed along each of these axes at an equal distance from C. If a flat screen is placed at L′, the proper focal distance of the lens, it is too

far for the rays B′ and A′. For all the rays to be equally focused the screen must be of spherical shape. If there are a number of small objects at slightly different distances, the different lengths of the incident rays are an added problem.

In Figure 79 it can be seen that when the screen is at L′ it is in position to receive a sharp image. If the screen is placed at L″ or L‴ the image must be blurred; but if a stop, S, is so placed that it

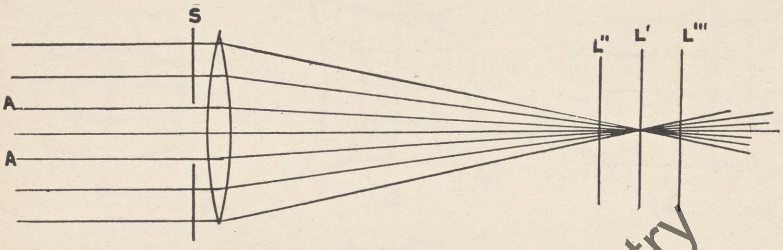


Figure 79.

allows only the axial rays, AA, to be utilized the blurring is made so slight as to be imperceptible. When the emergent rays subtend a small angle at the focus the position of the image-screen may be varied to a greater extent than if the angle is wide. This is what is known as *focal depth.*

Distortion. If an object composed of crossed lines (a, Figure 80) is viewed through a convex lens all the lines, except the central horizontal and vertical ones, appear curved and the whole object assumes the shape of a pin-cusion. This is caused by the increasing magnification as the rays approach the

edge of the lens and the corners of the square figure, being farther from the center than the centers of the sides, appear drawn out. The reverse is true of a concave lens and the figure appears barrel-shaped as in c.

Distortion cannot be overcome in any single lens, but it is less noticeable in the deep curved lenses than in the flat forms.

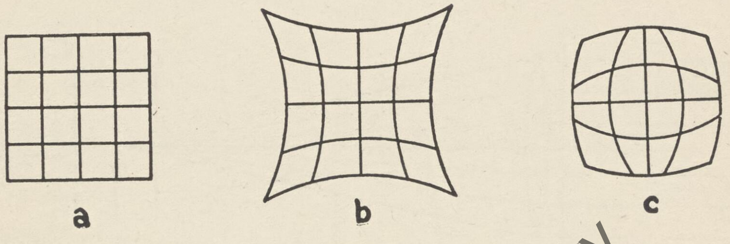


Figure 80.

Astigmatism of Oblique Pencils. When a pencil of light falls obliquely on a spherical refracting surface it becomes astigmatic and there will be two focal lines for a given object-point just as for a sphero-cylindrical lens.

In Figure 81, LL′L″ is a pencil of parallel rays obliquely incident at a spherical lens. LC is the axis of this pencil and it lies outside of the principal axis of the lens. The rays in the plane L′L′, striking the lens symmetrically at JJ′, will be focused equally at I, along the secondary axis LC. The rays L″O′ will be directed along O′SU and along OT.

Experiment. With the perforated cross for object and a 10 D. lens, form an enlarged image on a white screen. Place the object and image-screen on the extreme opposite ends of the bench. The clearest and brightest image of each luminous circle of the object will be surrounded by a halo of less intense light. Now with a stop, which obstructs the extreme marginal rays, the halo can be cut out.

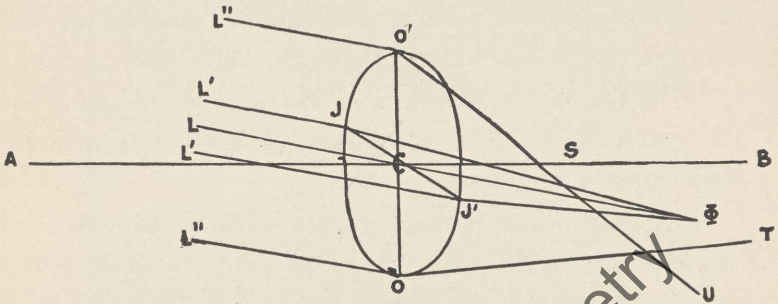


Figure 81.

Instead of the peripheral stop use a central one which allows only the marginal rays to pass through the lens. If the position of the screen is unchanged it will be seen that not only is the image diffuse, but there is also a play of colors due to dispersion by the prismatic action of the edge of the lens. If the screen is moved forward to the closer focal plane of the more refracting margin of the lens a clear image can be received, but the colors still remain.

Replace the image-screen in the first position at the end of the optical bench, and turn the lens slightly

around a vertical axis so that the light strikes the lens obliquely. The image will be seen to become elongated horizontally and, if the screen is now brought closer, a position can be found where the image line is vertical.

The spherical and chromatic aberrations, as well as distortion and curvature, in ordinary ophthalmic lenses, are not noticeable enough to constitute serious defects. They scarcely manifest themselves except in strong glasses, when the line of vision passes through them elsewhere than through the centers. It is in the latter position of the eye, in relation to the glass, that the astigmatism of oblique pencils is sometimes a serious problem.

An ophthalmic lens is of the best form when it has surfaces of different radii, the proportion depending on the power and the index of refraction of the glass. Tscherning has constructed a graph, called Tscherning's curve, giving the proper coflexure for rendering most lenses anastigmatic. These lenses are called point-focal. The vertex power of the lens is corrected with respect to a point at the center of rotation of the eye, which is fixed at a distance of 25 mm. from the lens (see page 180).

Lenses having different curves for every power are impracticable from the standpoint of the manufacturer, but lenses ground with base curves covering groups of powers can be made to sufficiently eliminate the astigmatic aberration. In the higher

powers the correction can only be obtained by aspherical surfaces.

In the diagram representing Tscherning's curve the abscissæ give the resultant dioptric powers of the

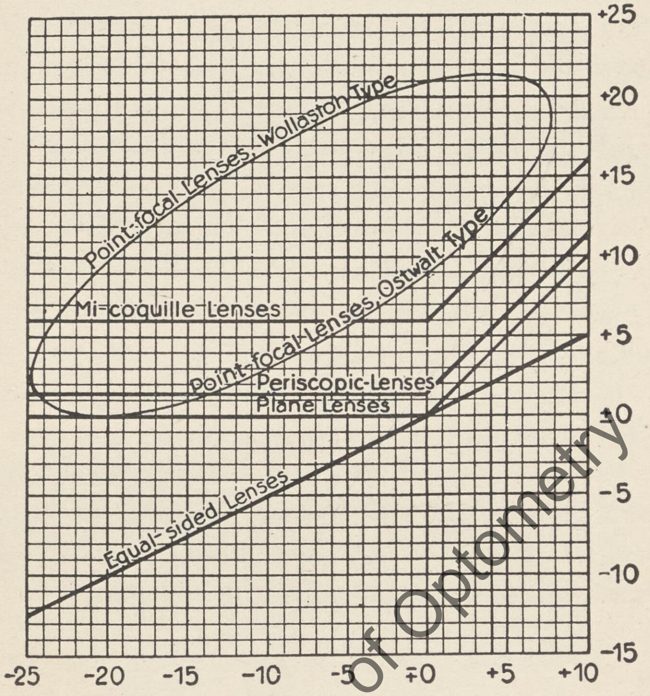


Figure 82.—Tscherning's curve (*Henker*).

lenses and the ordinates represent the powers of the front surfaces.

Suppose we desire to know the necessary curves for a plus 5 D lens. The vertical line corresponding to plus 5 will be seen to cut the ellipse, first at about 13.5 and again at about 21.3. If we choose 13.5 for

the power of the first surface the back surface will necessarily have such power that, allowing for thickness and curvature, the resultant lens will have a dioptric power of +5 D at the back surface. The deeper, so called Wollaston type, lenses are seldom used. The diagram also shows the older forms of commercial lenses.

A study of Tscherning's graph will show that the back surfaces of convex lenses from 0 to 7 D vary between —8.50 and —9 D; the back surfaces of the concave lenses from 0 to 4 D vary from —9.50 to —10.50 D. Generally speaking, it seems that an average constant base curve of —6., 7. or 8. D, for these powers will so nearly approach the ideal that it will fulfil the practical requirements of an ophthalmic lens.

It will be found in practice that the deeply flexed lenses, even though theoretically proper, are, in some cases, less comfortable to the patient than the older forms. This is especially true in myopia.

CHAPTER VII.

DIOPTRIC SYSTEM OF THE EYE.

The dioptric apparatus of the eye consists of the corneal system and the crystalline lens, which are nearly centered on a line passing through the anterior and posterior poles of the eye. This line is called the *optic axis of the eye.* The rays emerge in the vitreous and are imaged on the retina.

In order to locate the cardinal points along the axis, it is first necessary to apply the theory of Gauss to the corneal system, then to the lens and then to the combined system. We shall presume that the eye is in a state of rest, that is, its *static refraction.*

Corneal System. The cornea and aqueous constitute the corneal system. The refractive index of the aqueous, which is separated from the air by the cornea, is 1.336. It is more nearly optically empty than any other medium of the eye, and because the difference between the refractive index of air and that of the aqueous is greater than the differenc between any other two media, the cornea is the most important refracting surface of the eye.

The form of the anterior surface of the cornea resembles an ellipsoid of revolution. The antero-posterior axis is longest. The central portion may be considered spherical with a radius varying in

different eyes between about 7 and 8.5 millimeters. The normal average radius of the anterior surface of the cornea is taken to be 7.829 by Helmholtz, 7.98 by Tscherning and 7.7 mm. by Gullstrand.

The radius of curvature of the posterior surface is given as 6.5 by Tscherning and 6.7 mm. by Gullstrand. The radius of curvature of the posterior surface being greater than that of the anterior, the form of the cornea, therefore, is that of a convexo-concave or negative meniscus lens; but the effect of this is so slight that it may be disregarded and the substance of the cornea and the aqueous considered as one, the index of refraction being 1.336. The distance from the anterior surface of the cornea to the anterior surface of the lens is about 3.6 mm.

The formulæ for a single refracting surface are applied to the corneal system, and its principal focal distance in air or the *first focal distance,*

$F = \frac{n'-n}{r}$ or $F = \frac{n'-n}{\frac{1}{R}}$

$$F = \frac{R}{n - 1}$$

$f = \frac{1}{F}$

$r = \frac{1}{R}$

and the dioptric power on this side

$$D = \frac{n - 1}{R \text{ meters}}$$

If we assume the radius to be 7.8 mm. and the index 1.33

$$F = \frac{7.8}{1.33 - 1} = 23.66 \text{ mm.}$$

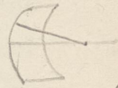
Assume radius 7.8 mm
$n' = 1.33$
$n = 1.00$
f distance = 23.66

and

$$D = \frac{1.33 - 1}{.0078} = 42.30 \text{ D}$$

cornea = ellipsoid
r = between 7 → 8.5 mm
Average r = 7.829 Helmholtz
" " = 7.98 Tscherning
" " = 7.7 Gullstrand.

The *second principal focal distance of the cornea* or that in the aqueous must be



$$F' = F \times n = 23.66 \times 1.33 = 31.46 \text{ mm.}$$

or

$$F' = F + R = 23.66 + 7.8 = 31.46 \text{ mm.}$$

Posterior radius
6.5 = r = Tscherning
6.7 = r = Gullstrand

and the dioptric power

Radius of curvature of posterior surface is > therefore we have a convexo concave or negative meniscus lens.

$f_{nd} = f'n'$

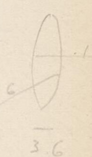
$$D' = \frac{D}{n} = \frac{42.3}{1.33} = 31.80 \text{ D}$$

These distances are measured from the single principal point of the system which lies on the axis at the intersection of the bounding surface.

Crystalline Lens. The lens has a biconvex form. Its anterior surface has a radius of 10 mm. in the center, and its posterior surface has a radius of about 6 mm. A thickness of 3.6 mm. along the axis is generally adopted. It is made up of a number of superimposed layers whose curvatures and indices of refraction increase from the periphery toward the center. The layers take the form of a number of menisci, surrounding a core or nucleus of high indicial power with surface curvatures of short radii.

$f' = f + \frac{1}{r}$

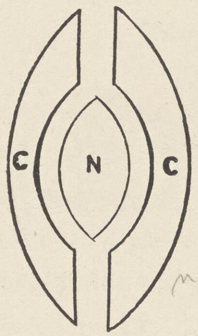
$D' = \frac{D}{n}$



As a result of its peculiar structure the lens has a higher dioptric power than if the refractive index were the same throughout. Consider the lens divided

into three parts as in Figure 83 where the biconvex nucleus, N, lies between two convexoconcave menisci, C and C. It can readily be seen that if the index of refraction of the menisci is increased they become more powerful as negative lenses and, therefore, able to neutralize more of the positive power of the core.

Supposing the lens to be homogeneous through-



n = 1.43

Figure 83.

out, its total index, as given by different investigators, varies between 1.406 and 1.455. It is surrounded by a single optical medium, since the index of refraction of the aqueous is the same as that of the vitreous. The principal points, therefore, coincide with the nodal points. If we assume that the total index of the lens is 1.43, the relative index of refraction of the lens in the aqueous and vitreous is

$$\frac{n'}{n} = \frac{1.43}{1.33} = 1.075$$

where n′ signifies the index of refraction of the lens and n the index of the aqueous and vitreous.

We proceed to find the focal power of each surface of the crystalline lens the same as with a glass lens in air, but with the value of the relative index of the lens in the aqueous and vitreous. The first principal focal distance, *i.e.*, in the aqueous, of the first surface is

$$F = \frac{R}{n - 1} = \frac{10}{1.075 - 1} = \frac{10}{.075} = 133.33 \text{ mm.}$$

where n represents the relative index of refraction of the lens. The dioptric power of the first surface on this side, designating the radius, R, in meters,

$$D = \frac{n - 1}{R} = \frac{.075}{.01 \text{ m.}} = 7.50$$

The second principal focal distance of the first surface, *i.e.*, in the substance of the lens,

$$F' = F \times n = 133.33 \times 1.075 = 143.33 \text{ mm.}$$

and the dioptric power for this surface on the same side

$$D' = \frac{D}{n} = \frac{7.50}{1.075} = 6.97 \text{ D}$$

Proceed in the opposite direction for the powers of the second surface, using the same formulæ. The first principal focal distance—the focal distance in the vitreous—

$$F = \frac{R}{n - 1} = \frac{6}{.075} = 80 \text{ mm.}$$

$$D = \frac{n - 1}{R \text{ meters}} = \frac{.075}{.006} = 12.50$$

The second principal focal distance of the second surface, the focal distance in the lens substance,

$$F' = F \times n = 80 \times 1.075 = 86 \text{ mm.}$$

and the dioptric power for this surface in the same direction

$$D' = \frac{D}{n} = \frac{12.50}{1.075} = 11.62 \text{ D}$$

The optical center lies on the axis where it divides the thickness of the lens (Figure 66) so that the relation

$$\frac{a}{b} = \frac{\text{radius of curvature of the first surface}}{\text{radius of curvature of the second surface}} = \frac{10}{6}$$

or

$$\frac{a}{b} = \frac{\text{F of the first surface}}{\text{F of the second surface}} = \frac{133.33}{80}$$

$$\frac{a}{b} = \frac{\text{F}' \text{ of the first surface}}{\text{F}' \text{ of the second surface}} = \frac{143.33}{86}$$

$$\frac{a}{b} = \frac{\text{D of the second surface}}{\text{D of the first surface}} = \frac{12.50}{7.50}$$

$$\frac{a}{b} = \frac{\text{D}' \text{ of the second surface}}{\text{D}' \text{ of the first surface}} = \frac{11.62}{6.97}$$

To find a we divide the thickness of the lens into $10 + 6 = 16$ parts and count off 10 from the first surface, that is to say that

$$a = \frac{tR}{R + R'}$$

and

$$b = \frac{tR'}{R + R'}$$

where a signifies the distance of the optical center from, and R the radius of, the first surface; b, the distance of the optical center from, and R′ the radius of, the second surface; t, the thickness of the lens along the axis (see Figures 66 and 67). F or F′ of the first surface, D or D′ of the second surface may be substituted for R if F or F′ of the second surface, D or D′ of the first surface are respectively substituted for R′ in the formulæ (see page 120).

The thickness of the crystalline lens being 3.6 mm.

$$a = 3.6 \times \frac{3.6 \times 10}{10 + 6} = 2.25 \text{ mm.}$$

and

$$b = 3.6 \times \frac{3.6 \times 6}{10 + 6} = 1.35 \text{ mm.}$$

Having found the point C on the axis, the image (JH, Figure 67) of the plane OC by the first surface may be found, as in the glass lens, by the formula $ll' = FF'$.

$$FF' = 133.33 \times 143.33 = 19110.1889$$
$$l = f - F = 2.25 - 143.33 = -141.08$$
$$l' = -135.45$$

The first principal plane of the crystalline lens, JH, lies 135.45 mm. from the first principal focus of the first surface in a negative direction—inside the lens 2.12 mm. from the first surface.

The image of OC by the second surface is found in the same way. Here

$$FF' = 80 \times 86 = 6880$$
$$l = 1.35 - 86 = -84.65$$
$$l' = -81.27$$

The second principal plane of the crystalline lens, J'H', lies within the lens at a distance, along the axis, 1.27 mm. from the second surface.

The principal focal distances, which are the same on each side, may be found by the same methods as were used with the convex lens (see pages 127-131).

The reduced thickness of the lens,

$$t = \frac{d}{n} = \frac{3.6}{1.075} = 3.34 \text{ mm.}$$

In the formula

$$\frac{1}{F' - t} + \frac{1}{F''} = \frac{1}{Fe}$$

the focal intercept, Fe, will be in the vitreous or aqueous so that it is necessary to reduce all values

to agree for these media, and the reduced thickness must be considered the distance between the two surfaces of the lens.

For rays parallel in the aqueous and, therefore, incident at the anterior surface of the lens, with our values for the crystalline lens, we may write, just as for the thick convex lens,

$$\frac{1}{133.33 - 3.34} + \frac{1}{80} = \frac{1}{49.52}$$

The focal intercept, that is, the distance from the posterior surface of the lens to the posterior principal focus is 49.52 mm. The distance from the second principal plane to the posterior or second principal focus—*the second principal focal distance of the crystalline lens*—

$$F' = 49.52 + 1.27 = 50.79 \text{ mm.}$$

To find the first principal focus we advance in the opposite direction. The rays are now parallel in the vitreous and incident at the posterior surface of the lens. The formula now is

$$\frac{1}{80 - 3.34} + \frac{1}{133.33} = \frac{1}{48.67}$$

Hence the anterior focal intercept is 48.67 mm. and the *anterior* or *first principal focal distance of the crystalline lens,*

$$F = 48.67 + 2.12 = 50.79 \text{ mm.}$$

Substituting our values in Gullstrand's formula for the equivalent focal power of a lens,

$$D = D' + D'' - t\,D'D''$$

we have for t D′D″

$$\frac{.0036 \text{ meters}}{1.075} \times 7.50 \times 12.50 = .31$$

and

$$7.50 + 12.50 = 20 - .31 = 19.69 \text{ D}$$

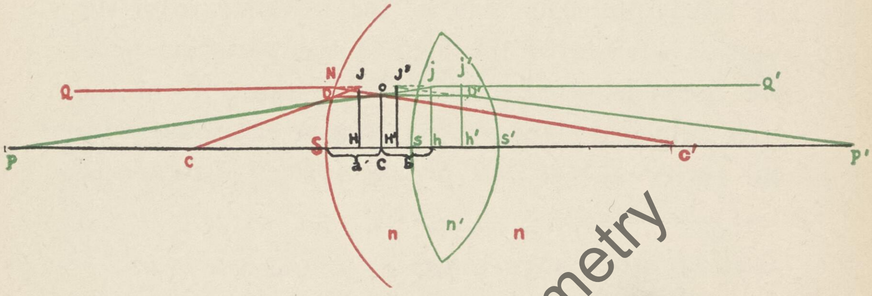


Figure 84.

Principal Points and Planes of the Eye. Let c and c′ be the principal foci of the cornea, S (Figure 84), p and p′ the foci of the lens, and h and h′ the principal points of the lens.

A ray QN, parallel to the axis, is refracted by the first surface, which is the principal plane of the cornea, toward c′. A ray coming from Q′, parallel to the axis, is directed from j, a point on the first principal plane of the lens, to p. Nc′ and jp cross at O. Drop a perpendicular, OC, to the axis.

The two pairs of similar triangles, c′NS and c′OC, pjh and pOC give

$$\frac{c'S}{SN} = \frac{c'S - SC}{OC}$$

and

$$\frac{ph}{SN} = \frac{ph - hC}{OC}$$

Call c′S, the posterior focal distance of the cornea, fc′; ph, the focal distance of the lens, fp; SC = a and hC = b. Then

$$\frac{fc'}{fp} = \frac{fc' - a}{fp - b}$$

and

$$\frac{a}{b} = \frac{fc'}{fp} = \frac{31.46}{50.79}$$

When one system is added to another the components are considered to be separated by the distance between their adjacent principal points. If we regard Sh = t, then

$$a = \frac{tfc'}{fc' + fp}$$

the same as for a convex lens. We know that t = Ss + sh = 3.6 + 2.12 = 5.72 mm. Substituting these values,

$$a = \frac{5.72 \times 31.46}{31.46 + 50.79} = 2.18 \text{ mm.}$$

and b, therefore, = 5.72 — 2.18 = 3.54 mm. This is the optical center of our eye.

Having located the point C, seek the principal points and planes of the eye. Let OC be an object. Determine the place where the images of it, furnished conjointly by the surface S and the crystalline lens, are formed.

The ray OD, parallel to the axis, is refracted toward c. The ray ON, directed from the posterior focus of the cornea, emerges parallel to the axis. The emergent rays, NQ and Dc, being divergent must be made to meet by projecting them backward to J. Hence J is the virtual image of O.

The ray OD′, parallel to the axis, meets the second principal plane of the lens at D′ and is refracted toward p′. Prolonged backward it meets the line Q′J′ at J′, and thus determines the image, J′, of O, furnished by the lens. Hence JH and J′H′ are the images of OC seen through the cornea and through the lens. They are erect and of equal size. The planes passed through JH and J′H′ are, therefore, the principal planes of the dioptric system of the eye.

The triangles c′NS and c′OC are similar, as are cJH and cDS, from which is derived

$$\frac{JH}{DS} = \frac{cH}{cS}$$

and

$$JH = NS$$
$$DS = OC$$
$$cS = fc$$
$$cH = fc + SH$$

Hence may be written

$$\frac{JH}{OC} = \frac{cH}{fc}$$

From the similar triangles c'SN and c'OC is derived in a similar way

$$\frac{SN}{OC} = \frac{J'H'}{OC} = \frac{fc'}{fc' - a}$$

and JH being equal to J'H', we have

$$\frac{cH}{fc} = \frac{fc'}{fc' - a}$$

and

$$cH = \frac{fc\ fc'}{fc' - a}$$

Substituting the values in the formula, the distance from the anterior focus of the first surface to the first principal point

$$cH = \frac{23.66 \times 31.46}{31.46 - 2.18} = 25.42 \text{ mm.}$$

It will be seen that this formula is exactly the same as

$$FF' = ll'$$

in which

$$cH = l'$$

The distance, therefore, from the cornea to the *first principal point* of this eye is

$$25.42 - 23.66 = 1.76 \text{ mm.}$$

In order to find the posterior or second principal plane, J′H′, use the similar triangles pjh and pOC, p′J′H′ and p′D′h′.

$$\frac{jh}{OC} = \frac{fp}{fp - b}$$

and if fp′ designate the posterior focal distance of the lens

$$\frac{J'H'}{D'h'} = \frac{H'p'}{f\,p'}$$

J′H′ = jh and D′h′ = OC, so that

$$\frac{H'p'}{fp'} = \frac{fp}{fp - b}$$

and

$$H'p' = \frac{fp'\,fp}{fp - b}$$

Substituting the values

$$H'p' = \frac{50.79 \times 50.79}{50.79 - 3.54} = 54.59 \text{ mm.}$$

which, as before, has the same value as l' in $ll' =$ FF′.

It was found that the second principal plane of the crystalline lens was 1.27 mm. in front of the posterior surface of the lens and, therefore, 5.93 mm. from the cornea. The second principal point of the eye lies 54.59 mm. in front of the posterior focus of the lens, which, being 50.79 mm. behind its principal plane, lies

$$50.79 + 5.93 = 56.72 \text{ mm.}$$

behind the cornea. The *second principal point* of this eye lies

$$56.72 - 54.59 = 2.13 \text{ mm.}$$

behind the cornea.

The two principal points and planes are situated in the anterior chamber, and the distance between them is

$$2.13 - 1.76 = .37 \text{ mm.}$$

The anterior principal focal distance of the eye is measured from the anterior principal plane and the posterior principal focal distance is measured from the posterior principal plane.

Second Principal Focal Distance of the Eye. The second or posterior principal focus, Φ′, of the eye is the point in the vitreous where rays, parallel before entering the eye, are united. The second principal focal distance, F′, is the distance from the second or posterior plane to the posterior principal focus, H′ to Φ′.

A ray coming from Q, parallel to the axis, is refracted by the cornea toward its second principal

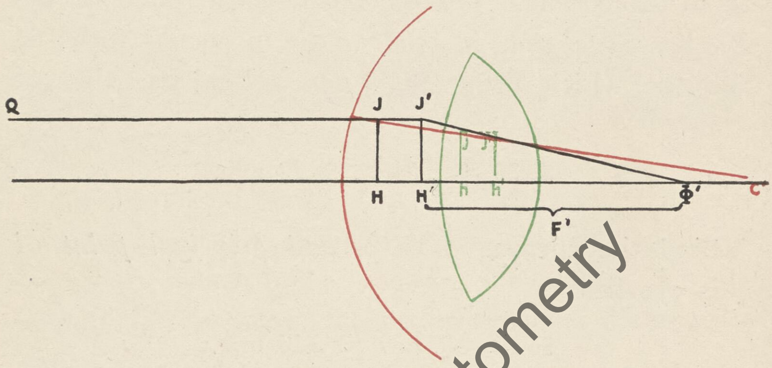


Figure 85.

focus, c′ (Figure 85). This ray meets the first principal plane of the lens at j and jj′ being parallel with the axis, we have in j′ one point in the path of the emergent ray. Another point in this path is given at J′ of the second principal plane of the optical system of the eye. Draw a straight line connecting J′ and j′ and the point on the axis where this line crosses is the second principal focus of the eye, Φ′.

To find the posterior principal focus of the eye by the method we have been pursuing, we must add the reciprocal of the principal focal distance of the lens system to the reciprocal of the focal distance of the corneal system, less the separation, *i.e.*, the distance between the two adjacent principal planes. This separation, t, is 5.72 mm. and, because the index of refraction of the medium between the components is the same as that in which the emergent rays are imaged, it must not be reduced.

We find that

$$\frac{1}{31.46 - 5.72} + \frac{1}{50.79} = \frac{1}{17.08}$$

The posterior focus is formed 17.08 mm. behind the second principal plane of the lens and this plane is 5.93 mm. behind the cornea. The distance from the cornea to the posterior principal focus of this eye is

$$17.08 + 5.93 = 23.01 \text{ mm.}$$

Since the second principal plane of the eye lies 2.13 mm. behind the cornea the *posterior principal focal distance* is

$$23.01 - 2.13 = 20.88 \text{ mm.}$$

With Gullstrand's formula we find the power to be

$$31.80 + 19.69 - .00572 \times 31.80 \times 19.69 = 47.90$$

diopters in the vitreous.

First Principal Focal Distance of the Eye. The first principal focus of the eye Φ (Figure 86) is the point at which all rays that were parallel in the eye are united after refraction. The first principal focal distance, F, is the distance between the first principal focus and the first principal point, H to Φ.

Let Q′j be a ray, parallel to the axis, in the eye. It meets the second principal plane of the lens at j′,

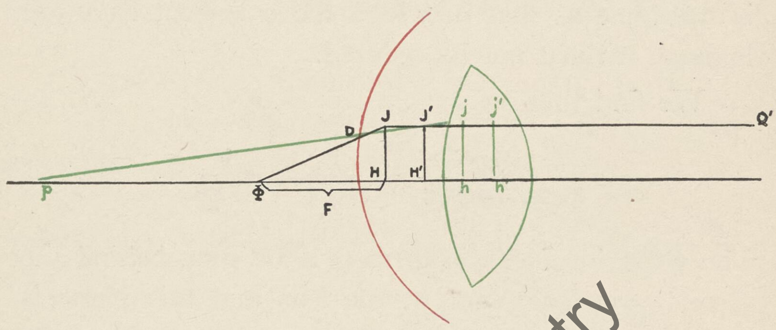


Figure 86.

the first principal plane at j, still parallel to the axis, and is refracted from this point toward the first focus of the lens, but it is again deviated at D by the cornea. The point D, at which it strikes the cornea, is a point of direction in the path of the emergent ray. Prolonging the ray Qj to the first principal plane, HJ, of the entire system another point, J, in the path of the emergent ray, is found. Connect J and D, and prolonging this line until it meets the axis, the *first principal focus,* Φ, of the eye is found.

To find the anterior principal focus by calculation we must be careful to reduce all values so that they agree for air before they can be correctly used in the formulæ.

The principal focal distance of the lens is 50.79 mm. in the vitreous and this must be reduced by

$$\frac{50.79}{1.33} = 38.18 \text{ mm.}$$

because a distance of 50.79 mm. in the vitreous is equivalent to 38.18 mm. in air. The same must be done for the distance between the adjacent principal planes and

$$t = \frac{5.72}{1.33} = 4.30 \text{ mm.}$$

The anterior principal focal distance of the cornea is, of course, unchanged.

To find the distance from the cornea to the anterior principal focus, using the reduced distances, we write

$$\frac{1}{38.18 - 4.30} + \frac{1}{23.66} = \frac{1}{13.93}$$

The first or anterior principal focus, Φ, of the eye being 13.93 mm. in front of the cornea, the *first* or *anterior principal focal distance* of the eye, F, is

$$13.93 + 1.76 = 15.69 \text{ mm.}$$

The values must be reduced in the same way for Gullstrand's formula. The dioptric power of the lens in vitreous becomes

$$19.69 \times 1.33 = 26.18$$

diopters for air;

$$t = \frac{.00572}{1.33} = .0043$$

the dioptric power of the cornea (42.30) remains. With these changes we write

$$26.18 + 42.30 - .0043 \times 26.18 \times 42.30 = 63.72$$

for the *dioptric power of the eye.*

Nodal Points of the Eye (Figure 87). Let T be a point in the first focal plane. A ray from it, parallel to the axis, pierces at J and J′ the two principal planes of the eye and is directed toward the second principal focus, Φ′; all rays emanating from T will, after refraction, be parallel to J′Φ′. Rays which, before refraction, are directed toward the first nodal point have, after refraction, a direction parallel to their primitive one and seem to come from the second nodal point. Hence a ray from T, parallel to J′Φ′, will be directed toward the *first nodal point* of the eye, K.

The ray TK strikes the first principal plane in h and necessarily appears to emerge from h′ in the second principal plane. If a line be drawn from h′, parallel to TK and J′Φ′, it will cut the axis in K′,

the *second nodal point* of the eye. KK′ is parallel to hh′ and hK is parallel to h′K′ so that

$$HH' = KK'$$

Since the difference between the first principal focal distance and the second principal focal distance of the eye,

$$F' - F = 20.88 - 15.69 = 5.19 \text{ mm.}$$

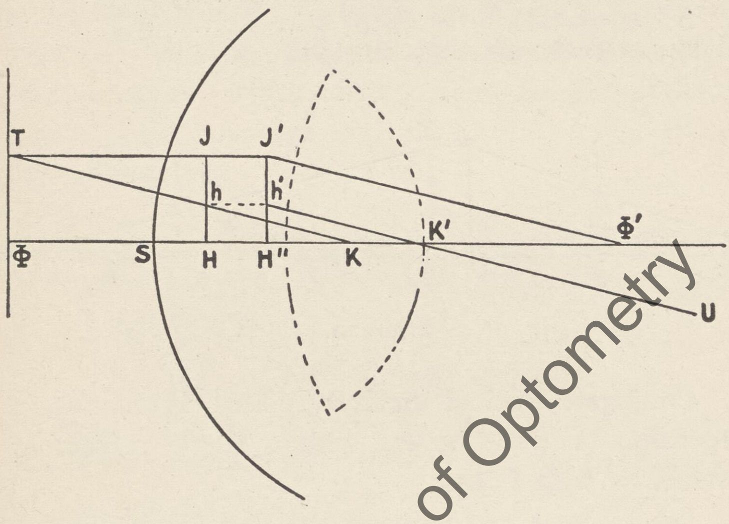


Figure 87.

the distance of the first nodal point from the cornea,

$$SK = SH + HK = 1.76 + 5.19 = 6.95 \text{ mm.}$$

The distance of the second nodal point from the cornea,

$$SK' = SK + KK' = 6.95 + .37 = 7.32 \text{ mm.}$$

The nodal points of the eye, therefore, lie very near the posterior capsule of the lens.

CARDINAL POINTS OF GULLSTRAND'S SCHEMATIC EYE RELAXED.

Length of eye	24	mm.
First principal focus from cornea	15.707	mm.
First principal point, SH	1.348	mm.
Second principal point, SH′	1.602	mm.
Second principal focus, SΦ′	24.387	mm
Anterior focal distance	17.055	mm.
and the posterior focal distance	22.785	mm.
therefore the power of this eye is	58.64	D

No importance is attached to the nodal points.

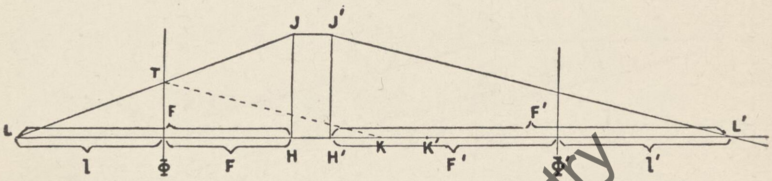


Figure 88.

Conjugate Foci of the Eye. This is almost a repetition of what occurs with a single refracting surface or with a lens.

Let LJ be a ray piercing the first focal plane at T and the first principal plane at J (Figure 88). The point J′ of the second principal plane, from which the ray in the last medium appears to arise, is found by drawing a line, parallel to the principal axis, from J to J′.

In order to determine the final direction of the ray in the last medium, draw the line TK toward

the first nodal point. Since T is situated in the first focal plane all rays emanating from this point are parallel to each other in the last medium, and parallel to the ray of direction which passes through the first nodal point. TJ may be considered another ray coming from T. Its final direction, J′L′, will therefore be parallel to TK. Hence the point L′, at which this ray meets the axis, is the image of L.

To find the conjugate foci by calculation use the same formula, FF′, $= ll'$, as for a single refracting surface. The only difference is in the absolute values of the lengths, which are not measured here from a single point, but from the two points, H and H′, as for a thick lens.

Example. Suppose an object is placed 250 mm. in front of our eye. Substituting the distances found in our calculations,

$$FF' = 20.88 \times 15.69 = 327.60$$

The object is 250 mm. in front of the eye and, therefore, $250 + 1.76 = 251.76$ mm. in front of H. Hence $f = 251.76$ and

$$l = f - F = 251.76 - 15.69 = 236.07 \text{ mm.}$$

and

$$l' = 327.60 \div 236.07 = 1.38 \text{ mm.}$$

The image is 1.38 mm. behind Φ′ or

$$20.88 + 1.38 + 2.13 = 24.39 \text{ mm.}$$

behind the cornea.

Size of the Image furnished by the Eye or any other Compound System. In order to find the image of an object (AL, Figure 89) find where the image of A is formed.

First draw the ray AJJ′ parallel to the axis and which, after having passed through the system, is directed toward the second principal focus, Φ′. Another ray which, before entering the system passes

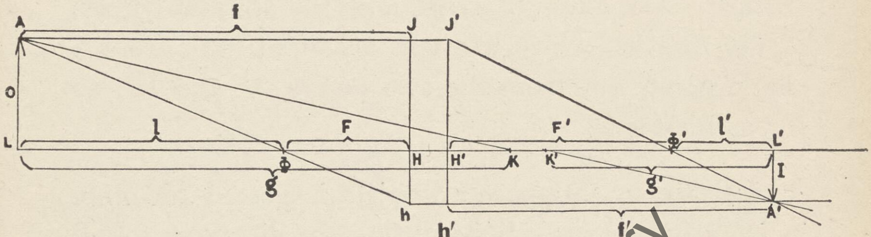


Figure 89.

through the first principal focus, Φ, strikes the first principal plane in h, emerges parallel to the axis, and cuts the ray J′Φ′ to form the image-point of A at A′. Still another line, AK, striking the axis in the first nodal point, appears to emerge from the second nodal point, K′, in a direction parallel to AK and crosses the first two lines in the image-point. The perpendicular dropped from A′ cuts the axis in the point L′ where the image of L is formed. Hence L′A′ is the image of LA.

The similar triangles ALK and L′A′K′, ALΦ and ΦHh, J′H′Φ′ and Φ′L′A′ give us the following formulæ:

$$\frac{I}{0} = \frac{K'L'}{KL} = \frac{g'}{g} = I = \frac{0g'}{g}$$

$$\frac{I}{0} = \frac{\Phi H}{L\Phi} = \frac{F}{l} = I = \frac{0F}{l}$$

$$\frac{I}{0} = \frac{\Phi' L'}{\Phi' H'} = \frac{l'}{F'} = I = \frac{0l'}{F'}$$

the same formulæ as those for a single surface and for a lens.

Emmetropia. Ametropia. The retina is in position to receive a clear image of a distant object when it lies in the plane of the posterior principal focus of the dioptric system. This condition is called *emmetropia.* The most distant point for which an eye is adapted when at rest is called its *punctum remotum* or *far point.* An emmetropic eye may, therefore, be defined as one whose far point is at infinity regardless of its length or dioptric power. The retina lies in the principal focal plane of the dioptric system and, therefore, it and a point at infinity are conjugate.

An eye which is not emmetropic is called *ametropic.* In ametropia the retina does not coincide

with the principal focus of the dioptric system. An eye is called *myopic* when the retina lies behind the principal focal plane, and *hyperopic* when it lies in front of the focal plane. That is to say, the dioptric power being the same, the myopic eye is longer and the hyperopic eye shorter than normal.

EXPERIMENT. Set up a 10. D lens on the optical bench and place a screen 10 cm. behind it to receive the image of a distant object. This may be

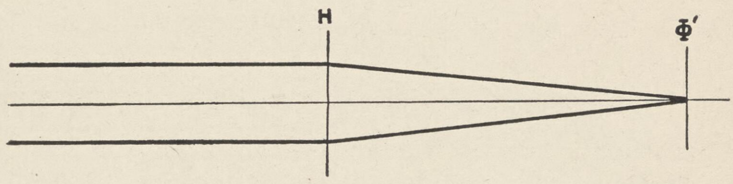


Figure 90.

considered an emmetropic eye in which the lens (H, Figure 90) is the dioptric apparatus and the screen at Φ′ is the retina.

If the screen is pushed forward or backward the image becomes blurred and, to render it clear, it is necessary to either increase or decrease the dioptric power.

It has been seen in experiments with lenses that the image may be substituted for the object and the object for the image. Applying this law to the emmetropic eye rays from a luminous point on the retina will be parallel after emerging from the eye. For

emmetropia (E) in which the punctum remotum (R) is at infinity we obtain the expression

$$E = \frac{1}{R} = \frac{1}{\infty} = O$$

Reduced Eye. To simplify calculations Donders constructed what is known as the reduced eye. In this eye, the radius of curvature of the cornea being 5 mm., there is no lens. Hence, the single nodal point lies 5 mm. behind the bounding surface and there is, as for any single refracting surface, one principal plane at the vertex of the cornea. The first principal focal distance is 15 mm. from the cornea and, since the index of refraction is 1.33, the second principal focal plane lies 20 mm. behind the cornea.

To agree with his schematic eye Gullstrand proposes a reduced eye having an index of refraction of 1.33 and a radius of curvature of the cornea of 5.7 mm.

A reduced eye which nearly agrees with most schematic eyes would have a radius of curvature of 5.5 mm. and an index of 1.33. In this the first principal focal distance is 16.5 mm. and the second principal focal distance is 22 mm. The dimensions of this eye may be used in the formulæ for a single refracting surface with results not greatly different from those obtained with the dimensions of most schematic eyes.

Angles Alpha and Kappa. Let AA′ (Figure 91) be the *optical axis* of the eye, which may be regarded as passing through the center of the cornea and the posterior pole of the eye. Generally, the axis of the cornea very nearly coincides with the axis of the eye. The optical axis of the eye contains the cardinal points of the entire optical system and also the *center of rotation,* C, which is situated on an average of about 13.7 mm. behind the apex of the cornea.

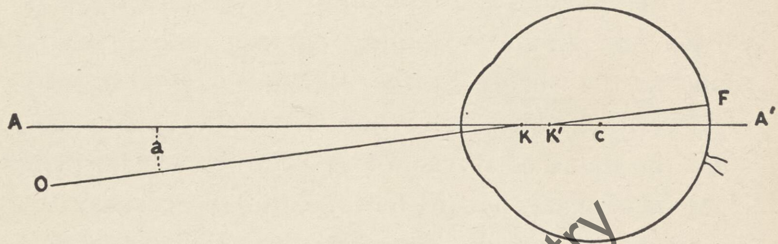


Figure 91.

The *line of vision,* OF, which connects the *point of fixation,* O, with the fovea, does not coincide with the optical axis. It has in common with the latter only KK′ where it crosses the optical axis. The visual line, therefore, is a secondary axis.

The angle OKA, formed by the optical axis and the visual line, is called the *angle alpha,* a. The angle a is called positive when the visual line lies to the nasal side of the optical axis, which is most frequently the case. When the optical axis and the visual line coincide there is no angle a; when the anterior portion of the optical axis lies on the nasal

side of the visual line the angle a is negative. The angle is very slight in the vertical plane.

The angle formed by the axis of the pupil and the visual line is called *kappa* by Landolt.

The summit of the anterior surface of the crystalline lens does not always coincide exactly with the corneal axis. Helmholtz and Knapp have found that the summit of the lens is sometimes situated as much as two degrees to the outer side of the corneal axis.

Gullstrand, while admitting that the optical axis of the eye may not exactly coincide with the optical axis of the lens, states that the deviations are so slight they cannot be used in calculations.

The center of the pupil is sometimes to the outer and sometimes to the inner side of the corneal axis.

The same aberrations exist in the eye as are described on pages 144-150 for an uncorrected convex lens, but of a less symmetrical nature. Owing to the structure of the retina these aberrations are ignored by the eye and do not interfere with distinct vision.

CHAPTER VIII.

MYOPIA.

In myopia the retina lies behind the principal focus of the dioptric system. The myopic eye is either too long or the refractive power is too great.

Myopia due to an increase in length is called *axial myopia.* When it is due to an increased re-

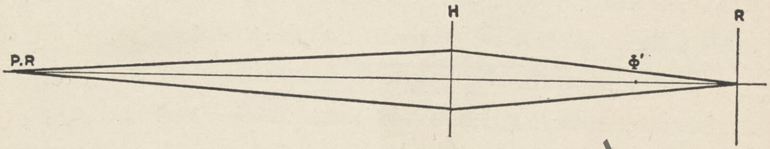


Figure 92.

fractive power it is called *curvature myopia.* Myopia may be caused by an increase of the index of refraction of the aqueous or lens, or by a decrease in the index of the vitreous—*myopia due to alteration of index of refraction.*

Axial myopia is the most common form. The others are rare and it is safe to assume that the dioptric power is constant and that the length of the eye varies.

EXPERIMENT. With the image screen, representing the retina (R, Figure 92) on one end of the optical bench, and on the other end the perforated cross

for the object, P. R., adjust a 10 D lens, H, so that a small image is distinctly in focus. Let this represent a myopic eye.

The object is at the P. R. or first conjugate focus, and the screen (retina) is at the second conjugate focus of this system. The second conjugate focal distance is, of course, greater than the principal focal distance (10 cm.) of our lens, because the object being at a finite distance the incident rays are divergent.

In order for parallel rays to be imaged on the

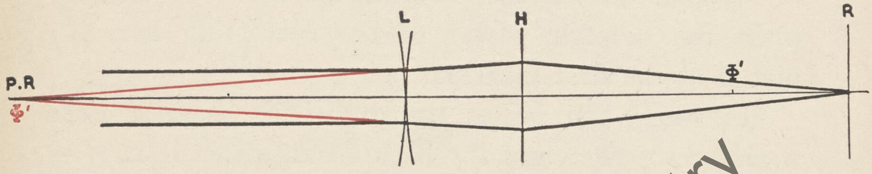


Figure 93.

retina of a myopic eye they must be made to diverge before entering the eye, as though they came from the punctum remotum, P. R. This will be done when a concave lens (L, Figure 93), of the proper strength, is so placed in front of the eye, H, that its second principal focus coincides with the P. R. Our myopic eye looking at a distant object through this lens, will see it clearly because the rays appear to come from the punctum remotum, and the lens is said to have corrected the refractive error.

An eye is more myopic in proportion as its retina

is situated farther behind the focus, or in proportion as its punctum remotum approaches the eye. The nearer the punctum remotum, the greater the myopia. If the far point is situated 25 cm. away the myopia equals $1/25$ cm. or $1/\frac{1}{4}$ m. or $1/.25$ m. or 4 D; if the distance is 50 cm. the myopia is $1/\frac{1}{2}$ m. or $1/.50$ m. or 2 D; if 1 m. the myopia is $1/1$ m. or 1 D.

To be exact, the distance must be measured from one of the cardinal points, preferably the anterior principal point of the eye. In this case the first conjugate focal distance, f, will represent the degree of myopia. If f is measured in meters, $1/f$ will express the dioptric power of the correcting lens if placed at the first principal point.

In weak, flat lenses the difference between the equivalent power and the back focal power is negligible. In the strong lenses and the deeply flexed forms the back focal power must be considered, but it is easy to calculate the effective power,

$$\frac{1}{Fe}$$

of the lens (pages 138-142).

The power of the correcting lens will depend upon the distance from the eye at which it is placed. The focal distance must be decreased and, therefore, the power increased as the lens is placed farther from the eye and nearer the P. R., since the focus of the lens must coincide with this point. In practice the lens is placed about 12 mm. in front of the cornea, so that

it is always stronger than the myopia which it corrects.

In Figure 94 it will be seen how, through a fixed spectacle lens, the rays reach the eye at different angles. When the eye is in the primary position (a) the optical axis of the lens coincides with that of the eye. In b it will be seen that not only is the oblique pencil, which leaves the lens, astigmatic, but the distance from the eye to the glass is greater.

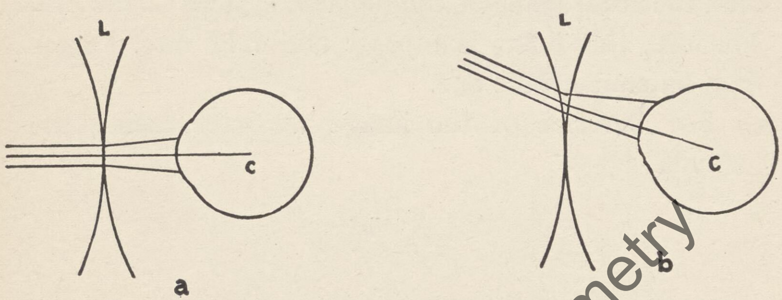


Figure 94.

In the very low refractive errors the effect of these oblique pencils or the changing distance between the lens and the movable eye, is scarcely noticeable. In the higher degrees, lenses as described on pages 150 to 152 are designed to almost completely correct these errors.

The Size of the Image in Myopia. Suppose the punctum remotum of a myopic eye is distant 200 mm. and an object, 3 mm. in size, is placed there. Accord-

ing to the formula $FF' = ll'$ we write, using the values of Gullstrand's schematic eye,

$$FF' = 17.055 \times 22.785 = 388.598$$

$$l = 200 - 17.055 = 182.945$$

$$l' = 388.598 \div 182.945 = 2.12 \text{ mm.}$$

which signifies that the image is formed 2.12 mm. behind Φ', or that the retina lies at this distance behind the focal plane of the dioptric system of the eye. The eye, therefore, is myopic and 2.12 mm. longer than an emmetropic one.

For the size of the image we have, using the formula

$$I = \frac{0F}{l}$$

$$I = \frac{3 \times 17.055}{182.945} = .279 \text{ mm.}$$

Using these same formulæ with the dimensions of the simplified eye described on page 179,

$$FF' = 16.5 \times 22 = 363$$

$$l = 200 - 16.5 = 183.5$$

$$l' = 363 \div 183.5 = 1.97 \text{ mm.}$$

and for the size of the image

$$I = \frac{3 \times 16.5}{183.5} = .269 \text{ mm.}$$

Consider another myopic eye whose far point lies at a distance of 100 mm. and consider an object 3 mm. in height at this place. Using the reduced eye we have

$$FF' = 16.5 \times 22 = 363$$

$$l = 100 - 16.5 = 83.5$$

$$l' = 363 \div 83.5 = 4.3 \text{ mm.}$$

and for the size of the image

$$I = \frac{3 \times 16.5}{83.5} = .59 \text{ mm.}$$

The size of the image in myopia, for an object at its punctum remotum, is greater than in emmetropia, and greater in proportion to the degree of myopia. The size of the retinal image will be larger in axial myopia than in curvature myopia, which can be readily understood after what has been studied about images in any system. In axial myopia the image of a distant object is the same size as in emmetropia; in curvature myopia it is smaller than in emmetropia.

The Effect on the Size of the Image by the Position of the Correcting Lens. In Figure 95 the point B of an object AB will be imaged somewhere along the axis BHB′. A ray, AΦ, passing through the anterior focus, Φ, will be parallel to the axis after refraction at H′. The image of AB will, of course, extend from a point in HB′ to another point, opposite to it, in H′A′. These lines being

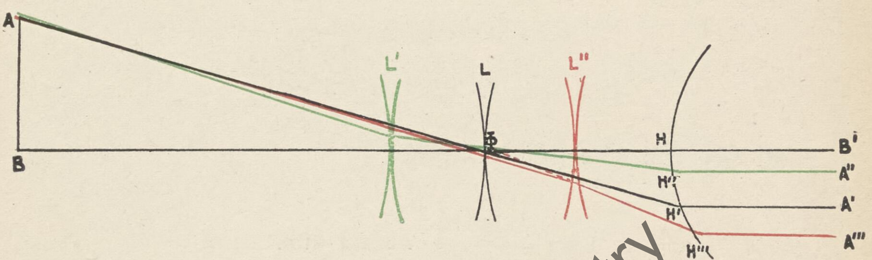


Figure 95.

parallel, the size of the image will be the same whatever the length of the eye.

If the correcting glass, L, is placed so that its optical center coincides with the anterior focus, Φ, of the eye the directions of the rays BB′ and AΦH′A′ will be unchanged, so that a lens placed in this position cannot affect the size of the image. That is to say, with the optical center of the correcting lens coinciding with the anterior principal focus of the eye, the size of the retinal image, in axial myopia, is the same as in emmetropia.

In practice the lens is generally so placed that its center is closer to the eye than its principal focal distance. This is especially true in concave menisci where the optical center lies outside the lens toward the eye. In the diagram is shown a ray from A so directed toward the lens L″ that, after refraction, it reaches the eye as though it came from Φ. After refraction at the eye it is parallel to the axis, as in the first case, but at a greater distance, and the image is larger.

Applying the same construction for the correcting lens, L′, placed farther from the eye than its anterior focus, it will be seen that the image is smaller.

From what has been said, not only need the correcting glass be weaker if placed close to the myopic eye, but the size of the retinal image becomes greater.

CHAPTER IX.

HYPEROPIA.

In hyperopia the retina is situated in front of the principal focus of the dioptric system of the eye. It is an eye which, in a state of rest, brings parallel rays to a focus behind the retina. In other words, the hyperopic eye is too weak to focus parallel rays on its retina.

In myopia the retina lies behind the focal plane, and rays, to be clearly focused, must come from an object at a finite distance, that is, they must diverge from a point in front of the retina. The far point of a myopic eye is within infinity. In emmetropia the retina coincides with the principal focus and only parallel rays will be focused on its retina—the far point of an emmetropic eye is at infinity. But in hyperopia the position of the retina is in front of the principal focal plane of the dioptric system, and rays, to be clearly imaged on the retina, must be convergent before entering the eye. The punctum remotum of a hyperopic eye is beyond infinity.

It is impossible to place an object beyond infinity, and convergent rays do not exist in nature; the rays can, however, be made convergent with a convex lens, and the far point of the hyperopic eye lies behind the eye at the place where these convergent rays would meet if continued backward.

The punctum remotum of a hyperopic eye is virtual, lies behind the retina, on the same side as its image and takes the negative sign, —P. R. As in myopia, hyperopia may be *axial, curvature* or *refractive,* or due to *alteration in index of refraction* (decrease in aqueous or lens, increase in vitreous).

EXPERIMENT. Set up a 10 D lens on the optical bench, and place the screen so that its distance is

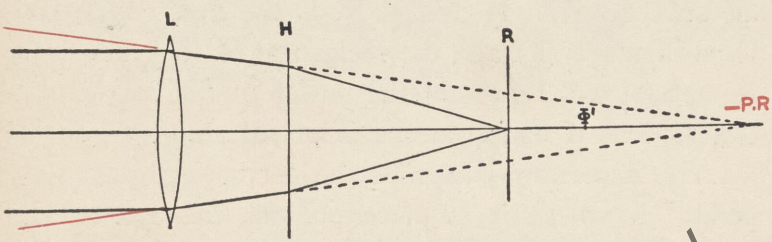


Figure 96.

less than the principal focal distance (10 cm.) of the lens. Let this represent a hyperopic eye. Find the convex lens which, added to the 10 D lens already in the carrier, will form a clear image of a distant object.

In order for parallel rays to be imaged on the retina of a hyperopic eye they must be made to so converge, before entering the eye, that they will meet, if prolonged, in the P. R. This will be done when a convex lens (L, Figure 96), of the proper strength, is so placed in front of the eye, H, that its second principal focus coincides with the P. R. The

hyperopic eye, looking at a distant object through this lens, will see clearly because the rays appear to come from a point beyond infinity.

As in myopia, an eye is more hyperopic in proportion as its retina is farther from the principal focus, or in proportion as its P. R. approaches the eye. If the distance between—P. R. and H is measured in meters the reciprocal of this, as in myopia, equals the degree of the hyperopia in diopters. The distance between H and P. R. is the first conjugate focal distance (f) and the reciprocal of this distance, in meters, will represent the power of the correcting convex lens if placed at the principal plane.

The necessary power of the correcting glass depends on where it is placed before the eye. The punctum remotum remains fixed, and, wherever the lens is placed, in order for it to correct the error, its second focus must coincide with this fixed point. Hence, when the lens is placed closer to the eye it is closer to the P. R. and its focal distance must be shorter. In contradistinction to the correcting glass in myopia, the correcting lens in hyperopia needs to be weaker as it is placed farther away from the eye. The lens is always weaker than the hyperopia which it corrects.

Conjugate Foci of a Hyperopic Eye. In Figure 97, QJ is a convergent incident ray which strikes the first principal plane in J. Because, if prolonged, it will cut the axis in L we consider L a virtual

object-point which takes the negative sign. Suppose T, in the focal plane, a luminous point, all rays from this point will then be parallel to each other after refraction. If we draw a line from this point to the first nodal point, K, it will leave the system as though it came from the second nodal point, K′, in a direction parallel to its first. The final direction of TJ will be parallel to the line of direction TK, but from J′ on the second principal plane. J′L′, therefore, is the final direction of the refracted ray. L′ on the axis is the image of the object L, in this case the object is virtual and the image is real.

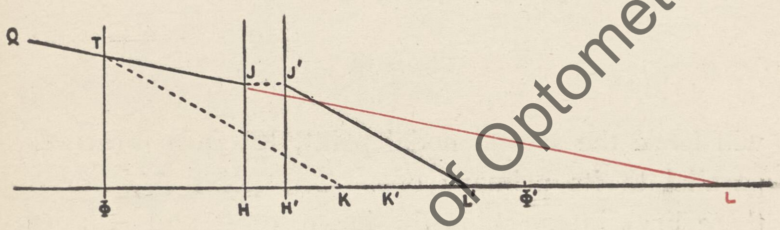


Figure 97.

The Size of the Image in Hyperopia. Having found L (Figure 98) erect AL and let it be the virtual object, O. The ray QA, parallel to the axis, will be directed to Φ′ from J′ on the second principal plane. The ray ΦA, directed from the first principal focus, Φ, striking the first principal plane in P will be refracted through P′ and be parallel to the axis. Another ray, KA, directed to the first nodal point,

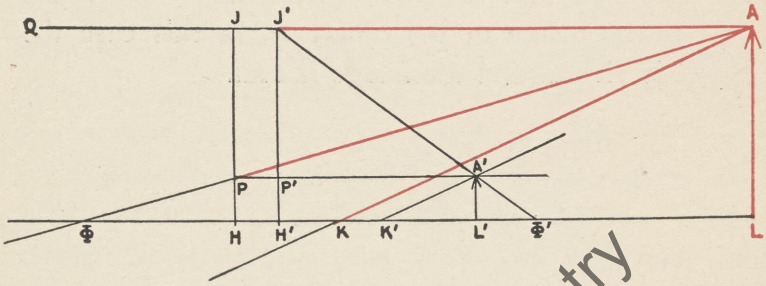


Figure 98.

will leave the second nodal point, K′, in a direction parallel to its primary one. The point A′, where these lines intersect, is the image of A and A′L′ is the image of AL.

The same formulæ are used in this as in myopia, but it must be remembered that f and l are negative. Suppose the object, 3 mm. in height, is situated 200 mm. behind the principal plane. According to the formula FF′ = ll', and with the values of the reduced eye,

$$FF' = 16.5 \times 22 = 363$$

$$l = -200 - 16.5 = -216.5$$

$$l' = 363 \div -216.5 = -1.67 \text{ mm.}$$

This means that a hyperopic eye, whose far point is 200 mm. behind the principal plane, is 1.67 mm. shorter than an emmetropic eye.

For the size of the image

$$I = \frac{0 \times F}{l} = \frac{3 \times 16.5}{216.5} = .228 \text{ mm.}$$

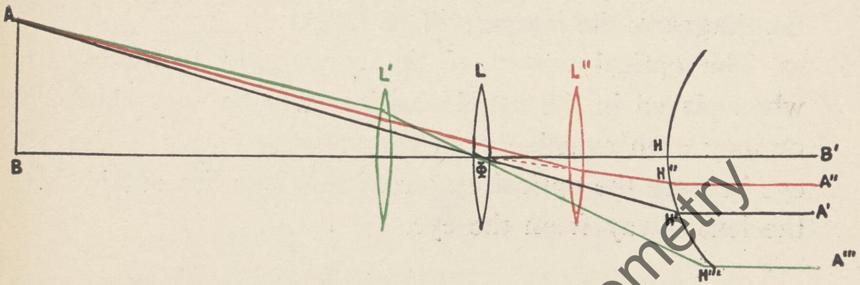


Figure 99.

The Effect on the Size of the Image by the Position of the Correcting Lens. In Figure 99 the image of a point, B, will be formed somewhere along the axis. The ray AΦH', striking H' as though it came from the principal focus, Φ, will be parallel to the axis after refraction. Regardless of where the image is formed, its size will extend from some point along the axis to a point opposite along H'A'.

As in myopia, if the correcting glass is so placed that its optical center coincides with the anterior

principal focus the size of the image in axial hyperopia will be unchanged and it will be the same size as in emmetropia. In refractive hyperopia the image is larger than in emmetropia.

If the convex lens is placed closer to the eye (L″) the ray AH″ having been bent toward the axis, and then refracted parallel to it (H″A″), lies closer to the axis and the image is smaller.

The reverse is true when the convex lens, L′, is placed at a greater distance from the eye than the anterior focus is. In this case, as can be seen in the diagram, the image will be larger.

The optical center of the convergent meniscus, when placed in the usual position, will be very close or may even coincide with the anterior focus of the eye because the optical center of this lens lies outside the lens, away from the eye.

CHAPTER X.

ASTIGMATISM.

Ametropia is divided into two opposite conditions —myopia and hyperopia. Sometimes, however, the refraction in the several meridians of the same eye is different. In one meridian the eye may be emmetropic and in another it may be ametropic, or, all meridians being ametropic, there may exist a difference in degree. This is called *regular astigmatism.*

When there is an unequal curvature in any one meridian the condition is called *irregular astigmatism.* Irregular astigmatism is probably, to a certain degree, present in every eye. The seat of this condition is usually in the lens; but it may be caused by some deformity or pathological deviation of the surface of the cornea. It cannot be corrected with lenses and will not be discussed in this place.

Regular astigmatism is present in all eyes. Donders considered that any degree up to .75 diopters may not be called abnormal; but a much lower error of astigmatism than this, however, may cause symptoms of asthenopia and should be corrected.

EXPERIMENT. Set up a combination on the optical bench, consisting of a spherical and cylindrical lens (plus 6 D sphere with plus 3 D cylinder) with axis of the cylinder at 180 degrees (horizontal) and

place it at about 75 cm. from one end of the bench. At the farthest end place the perforated cross, illuminated, as the object. In this system rays of light will be more sharply refracted in the vertical meridian than in the horizontal. This is the case in the majority of astigmatic eyes, and is called *astigmatism with the rule.*

Set up an image-screen very near the lens; gradually increase the distance and closely watch the

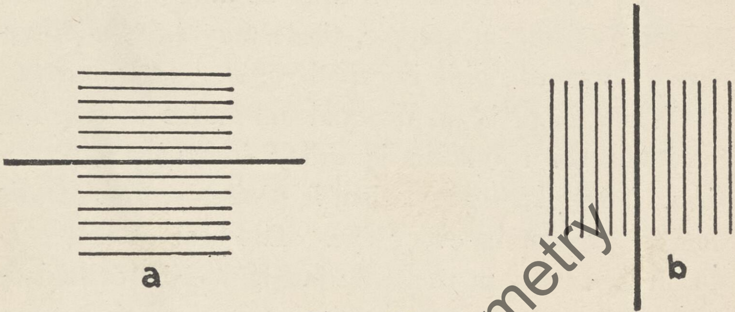


Figure 100.

form of the image of each luminous point of the cross. Close to the lens, if round, each perforation of the cross forms a circle of diffusion. As the screen is withdrawn each circle becomes flattened horizontally, forming an oval and, finally, at the *first focal plane* of the combination, the image of the cross will be seen as in Figure 100, a. It is here that each point of light forms an image which is a horizontal line.

As the screen is farther removed each horizontal line widens into a horizontal oval and then into a

circle of diffusion. This is the place in Stürm's conoid where there is least confusion. It divides the interfocal distance into two parts which are proportional to the focal distances of the components of the system.

On withdrawing the screen still farther the circle becomes longer in the vertical meridian, becoming narrower and narrower, until a place is reached where the image of each point is a vertical line. This is the *second focal plane* of the combination and the image of the whole object appears as in b, Figure 100.

Simple Myopic Astigmatism. The last arrangement, considering that the incident rays are parallel, may be taken to represent an eye in which the horizontal meridian is emmetropic while the vertical meridian is myopic. This eye, while at rest, sees vertical lines more plainly than horizontal ones. That is, although a vertical line is elongated it is nevertheless a line, while a horizontal line is broadened into a haze. The retina, in simple myopic astigmatism, is in the second focal plane of the dioptric system.

In our experiment the power in the horizontal direction is 6 diopters and its principal focal distance is about 16.6 cm. At this place is the screen which represents the retina, *i.e.*, the screen would be placed here if the object were at infinity.

The principal focal distance in the vertical direction, having a power of 6 D plus 3 D = 9 D, is about 11.1 cm. For rays to be focused at a distance

of 16.6 cm., by a lens with focal distance of 11.1 cm., it is necessary for the light to come from its conjugate focal distance which, in this case, is 33.3 cm. in front of the lens.

The lens necessary to correct this error would have to make parallel rays (in the vertical meridian only) appear to diverge from a point 33.3 cm. in front of the system. This would be the concave cylinder with its axis horizontal, which, placed anywhere in front of the system, has its posterior principal focal plane coincide with a point 33.3 cm. in front of the system. This lens neutralizes the excess power which existed in every meridian except the horizontal.

In simple myopic astigmatism the axis of the correcting concave cylinder must coincide with the emmetropic meridian of the astigmatic eye. When the axis lies less than 45 degrees from the horizontal meridian the astigmatism is said to be *with the rule.*

Compound Myopic Astigmatism. When the retina lies behind the second focal plane of an astigmatic eye parallel rays, in order to be focused on the retina, must first be made to diverge in all meridians, varying from the minimum in the plane corresponding to the axis, to the maximum along the meridian perpendicular to the axis. In this case the condition is first made into a simple myopic astigmatism with the proper concave spherical lens and the remaining astigmatic error then corrected by a concave cylinder.

Simple Hyperopic Astigmatism. In this form of astigmatism the retina lies in the first focal plane of the system. In our experiment the dioptric power in the horizontal meridian is 6 and in the vertical, $6 + 3 = 9$ D. This eye, in a state of rest, sees horizontal lines more plainly than vertical ones. The retina lies in the focal plane of the 9 diopter power and is emmetropic for this meridian. The power in the horizontal plane is 3 diopters weaker than in the vertical, and the rays in every meridian except the vertical come to a focus behind the retina. The conjugate focus, in the horizontal meridian, is therefore negative and lies behind the eye at a distance of 33.3 cm.

To make this eye bend parallel rays to a focus on the retina it would be necessary to make its refractive power equal to a convex spherical lens of 9 D. This can be done by adding a convex cylindrical lens which, with its axis at 90 degrees, will add 3 D. of power in the horizontal meridian. The lens, placed somewhere in front of the system, would need be of such strength that it would cause parallel rays in the horizontal meridian to converge to a point 33.3 cm. behind the eye.

In simple hyperopic astigmatism, as in simple myopic astigmatism, the axis of the correcting cylinder must coincide with the emmetropic meridian of the eye. When the axis of the correcting cylinder lies within 45 degrees on either side of the vertical or

90th meridian the hyperopic astigmatism is said to be with the rule.

Compound Hyperopic Astigmatism. This is the case when the retina is placed in front of the shortest focal distance of an astigmatic eye. The ametropia, as in compound myopic astigmatism, is first changed into a simple hyperopic astigmatism with a convex spherical lens, and the remaining astigmatism then corrected with a convex cylinder.

Mixed Astigmatism. When the retina is somewhere between the focal lines it is possible to correct the spherical error with either a convex or a concave spherical lens. The simple astigmatism which is left is myopic if a convex spherical is used, and hyperopic if a concave sphere is employed. The glass necessary to correct this error is a mixed cylinder.

All formulæ for emmetropia and ametropia may be used for each meridian in the calculations in astigmatism.

The meridians of greatest and least refraction are called *principal meridians.* In our experiments they are the vertical and the horizontal meridians. The maximum and minimum refraction lie in the principal meridians, which are perpendicular to each other. The degree of the astigmatism is expressed by the difference in refraction that exists between the two meridians.

An astigmatic eye never sees a point as a point; it is always a diffusion image. The way objects ap-

pear to such an eye depends upon the way a point appears. If either of the principal meridians is adapted to a luminous point the latter forms upon the retina a line of diffusion or focal line, which is perpendicular to the adapted meridian and parallel to the non-adapted one. If the focus of one of the intermediate meridians falls upon the retina the image of the point is diffused.

A line appears distinct to an astigmatic eye when it is parallel with the meridian which is not adapted to its distance, because, in this case, the elongated images of diffusion, of all the points composing the line, coincide with the image of the line itself. The line is seen indistinctly when it has the same direction to the meridian which is adapted to its distance; when diffusion images of its different points are perpendicular to the direction of the line, making it appear broadened and diffuse.

The inequality of refraction in the different meridians of the eye, which constitutes regular astigmatism, is due to anomalies of curvature in its refractive surfaces, or to eccentricities of these surfaces, or to both. The cornea is the most common seat of astigmatism.

The lens often is at fault, sometimes in a passive, at other times in an active way. If the lenticular astigmatism is parallel to the corneal astigmatism the total astigmatism is the sum of both; but this is not often the case. The greatest curvature of the lens is

usually in the horizontal direction, so that the lenticular astigmatism may partially or wholly compensate that of the cornea or even exceed it.

A difference in two mutually perpendicular meridians is produced when the eye looks obliquely through a spherical lens or, in other words, when the lens is not centered in regard to its axis. This also occurs when the dioptric system of the eye is out of center, especially where there is an inclination of the crystalline lens. There may be an astigmatism (usually against the rule) of the posterior surface of the cornea.

It must be remembered that the distance of the correcting lens from the eye may make it necessary for the concave cylinder to be stronger and the convex glass weaker than the real astigmatism.

Ophthalmometry. The measurement of corneal curvature is called ophthalmometry. For all practical purposes, the same principles and formulæ which apply to a convex reflecting surface may be applied to the cornea.

The general formula for mirrors is

$$ll' = FF$$

and for the size of the image

$$\frac{O}{I} = \frac{l}{F}$$

or

$$\frac{O}{I} = \frac{2l}{R}$$

and

$$R = \frac{2lI}{O}$$

which is the formula used in ophthalmometry (see pages 58-59).

If we know O, the size of the object; I, the size of the image; and *l*, the distance of the object, we can easily find R, the radius of curvature of the cornea.

The radius being known, the anterior focal distance is found by the formula

$$F = \frac{R}{n - 1}$$

or the power in diopters by

$$D = \frac{1000\ (n - 1)}{R}$$

(page 72).

The instrument used for these measurements is called an *ophthalmometer*. The ophthalmometer was invented by von Helmholtz who first described it in 1854. The improvements of Javal and Schiötz have made it possible for clinical use.

The fundamental parts of the modern ophthalmometer are a telescope, which carries a double refracting prism besides the objective and eye-piece, and two mires which slide on a graduated arc so that the distance between them can be varied and the arc

rotated into any meridian which is to be measured. The distance separating the mires serves as the size of the object.

We may either keep the mires fixed and measure the distance between their images, or we may separate the mires until there is a fixed distance between their images. The latter method is the one used in ophthalmometry. The fixed size of the image is accomplished by *doubling* with a birefracting prism of

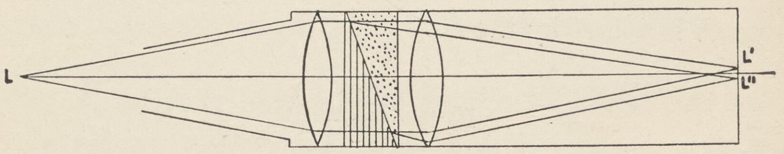


Figure 101.

quartz (Wollaston prism). By this doubling four images are seen instead of two.

By moving the mires along the arc the size of the object is made to vary until it corresponds with the doubling of the prism. This doubling is constant and in exact line with the plane of the graduated arc. The telescope, by magnifying the images, makes the reading easier. Figure 101 shows the position of the prisms in the telescope of the instrument. Two images of L are formed, one at L′ and the other at L″.

The eye to be examined is placed at the center of the arc and the instrument is adjusted so that the

images of the mires are plainly and clearly focused in the center of the cornea.

Let A and B (Figure 102) represent two objects which, having been doubled, are made to appear as four—A, B, A′, B′. It can be seen that, since A′, is the double of A and B′ the double of B, in order for B and A′ to coincide the doubling (t′) must be exactly equal to the separation between the two objects (t).

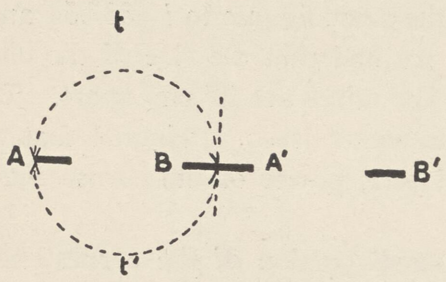


Figure 102.

The doubling of the images amounts to about 3 mm. and when the mires are so adjusted that the edges of their images coincide the image is exactly 3 mm. in size.

Javal and Schiötz have taken as the index of refraction of the cornea 1.3375, selected so that 45 diopters would correspond exactly to a cornea of 7.5 mm. radius.

By the formula

$$D = \frac{1}{F} = \frac{n - 1}{R} = \frac{.3375}{R}$$

if R is expressed in meters; and if expressed in millimeters

$$D = \frac{1000 \times .3375}{7.5} = \frac{337.5}{7.5} = 45$$

Placing the value of R in the formula,

$$R = \frac{2lI}{0}$$

and giving the same figures to l (337.5) and making $I = 3$ mm., we find that the size of the object is 27 cm., that is the mires are 27 cm. apart. The arc of the ophthalmometer is so graduated that either the radius or dioptric power of the cornea may be read off.

Only a small portion of the cornea, near where it is cut by the visual line, is measured. This measurement may be made in any meridian and if the cornea is spherical the measurements, of course, will be the same in all meridians. If the curvature is greater in one meridian than in another we have astigmatism, and in this case there would be two principal meridians at right angles to each other, one of which would have the greatest radius and the other the least. The difference between these powers is the amount of astigmatism, the axis of which will lie along the radius of least curvature. If there is irregular astigmatism the images will be distorted.

In Figure 103, A shows the images in alignment along a principal meridian and in contact; B, the images overlap, but still lie in a principal meridian; C, the images overlap and are off a principal meridian.

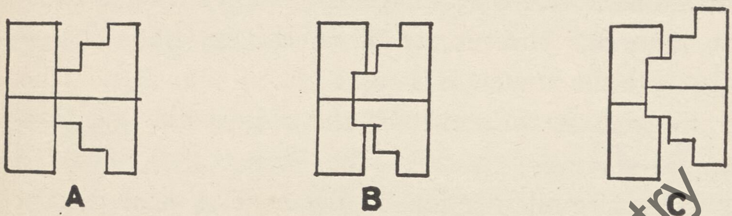


Figure 103.

CHAPTER XI.

ACCOMMODATION.

It has been seen that, with a convex lens, a distant object-point will be imaged as a point in the second principal focal plane. If, without changing the position of the image screen, the object is brought closer to the lens the image is formed behind the screen and, instead of a clear image, a section of the luminous cone is found on the screen. This is exactly what occurs in an eye.

Objects at different distances from the eye cannot be distinctly seen at the same time. Hold a finger before one eye and look past it at some distant object. When the distant object is distinctly seen the finger is blurred; when the finger is distinctly seen the distant object is blurred. Note that a certain effort is felt when changing the focus from the far to the near object.

Experiment. Place an image screen on one end of the optical bench. Set up a 10 D lens 10 cm. in front of the screen. Let this represent an emmetropic eye at rest. In an emmetropic eye the retina coincides with the principal focal plane of the dioptric system; the far point or punctum remotum lies at infinity and only distant objects can be imaged on the retina.

Now arrange a luminous object on the other end of the bench. A diffusion image will be perceived on the screen. In order to clearly image this near object on our screen it is necessary to increase the dioptric power of the system. This is done by adding the proper convex lens to the 10 D lens already in the carrier.

Just as in our set-up the eye cannot distinguish near objects when it is adapted for far, nor distant objects when it is adapted for near. In any dioptric system the image is distinct but for one single distance of the object. In order for the eye to see objects distinctly at different distances a change must be made, either in the position of the retina or in the refractive power of the dioptric system. This change takes place in the refractive power by a change in the convexity of the crystalline lens and is called *accommodation.*

In a state of rest the dioptric system of the eye presents its minimum of refractive power; the eye is adapted to the most distant point that it can see distinctly, *i.e.,* its punctum remotum. During a condition of greatest possible accommodation the dioptric system has acquired its maximum of refractive power and the eye is adapted to its nearest point—*punctum proximum.*

The total accommodative power which an eye possesses is represented by the difference between its refraction when at rest and the refraction when un-

der its maximum effort of accommodation. This has been called the *amplitude of accommodation.*

The refraction of the eye is inversely proportional to the distance of the object which it sees distinctly; or, the refraction must be stronger in proportion as the object is nearer.

Let R (Figure 104) represent the distance of the punctum remotum, P. R., from the eye. The refraction, r, at rest, is

$$r = \frac{1}{R}$$

This is the minimum or *static refraction* of the eye.

If P is the distance from the punctum proximum, P. P., to the eye the refraction, p, in a condition of maximum accommodation, is

$$p = \frac{1}{P}$$

The difference between the minimum refraction and the maximum refraction of an eye is

$$\frac{1}{P} - \frac{1}{R}$$

or, if the distance is expressed in meters,

$$p - r \text{ diopters}$$

The accommodation has the same effect as a convex lens added to the eye. An eye devoid of accommodation is adapted to its far point, P. R., and

will not be able to see anything nearer than that. To render vision possible, at the distance P, the rays must be made less divergent. This effect is obtained by adding the convex lens, L, which so changes the direction of the rays that, when they enter the eye, they appear to come from P. R. This lens adds the necessary power to the eye and, therefore, expresses the amplitude of accommodation in diopters.

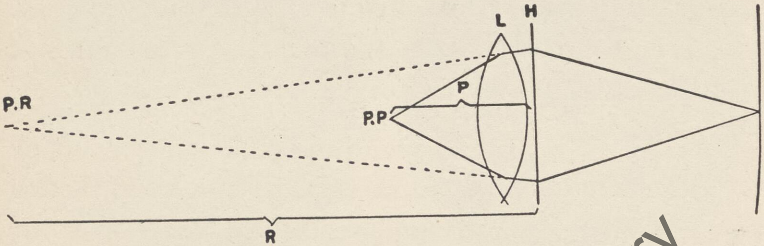


Figure 104.

In order to be exactly comparable to the amplitude of accommodation the lens ought to coincide with the first principal point, H, of the eye. Donders reckons from the first nodal point and Duane from the first principal focus of the eye.

If the focal distance of the auxiliary lens is repsented by A

$$\frac{1}{A} = \frac{1}{P} - \frac{1}{R}$$

which is the formula given by Donders to express the amplitude of accommodation.

If the distances A, P and R are given in meters we obtain the powers in diopters. In this case, for the amplitude of accommodation,

$$a = p - r$$

in which a represents the power or amplitude of accommodation in diopters; p equals the power, in diopters, of the eye when adapted to its punctum proximum, that is, its maximum refraction; and r, the power of the eye adapted to its punctum remotum.

The distance between the punctum remotum and the punctum proximum of an eye has been called, by Donders, the *region of accommodation.*

EXPERIMENT. Hold a lighted candle before an eye, so that it forms an angle of 30 degrees with the visual line, and observe from the other side at the same angle. Three small images of the light will be seen in the pupil (Figure 105). These are the reflected images by the cornea and the anterior and posterior surfaces of the lens. A represents their appearance when the eye is at rest, B their appearance in the accommodated eye.

The image a, nearest the flame, is that furnished by the anterior surface of the cornea. It is the brightest because the difference between the refraction of the cornea and air is greater than between the aqueous and the lens or the vitreous. It is the medium image in point of size because the curvature of the cornea is greater than that of the anterior sur-

face and less than that of the posterior surface of the lens. Since it is formed by a convex reflecting surface it is erect and virtual, and if the flame is moved the image will be seen to move in the same direction.

The image b is due to the anterior surface of the crystalline lens. This image is also virtual and erect.

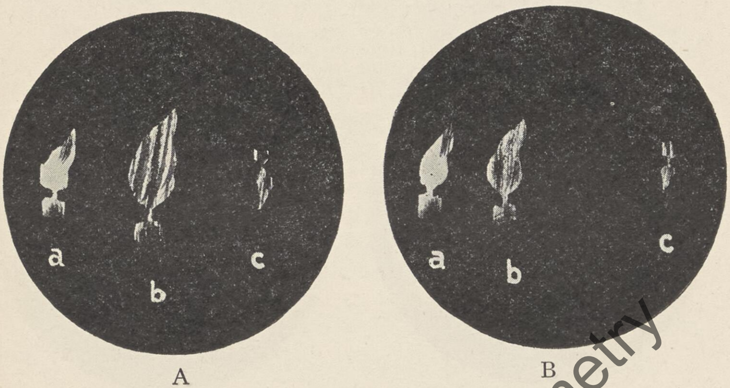


Figure 105.

It is largest because the surface which produces it is least convex and also because it is magnified by the cornea and the anterior chamber.

The image c, by the posterior surface of the crystalline lens, is smallest and least luminous. This image is formed by a concave reflecting surface and is, therefore, real and inverted. If the flame is moved the image will move in an opposite direction. It is small because it is reflected by the surface of greatest curvature, and least luminous because the light passes through thick layers of refractive media.

The three images here described are called the *images of Purkinje,* after the scientist who first discovered them. A fourth image, from the posterior surface of the cornea, has been described by Tscherning. In order to study these reflexes the room must be completely dark, except for the luminous object. Helmholtz used, instead of the lighted candle, a screen with two openings which were illuminated

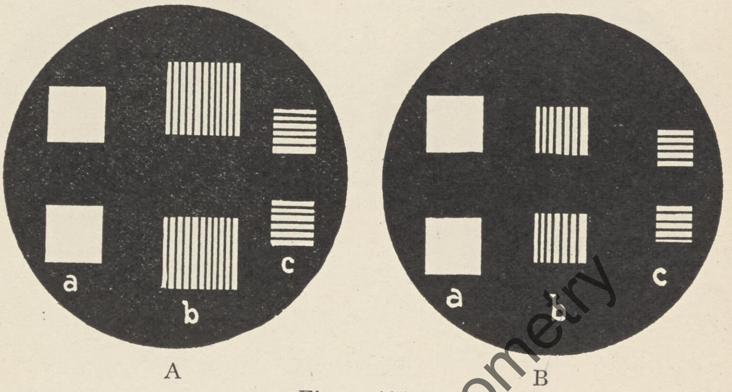


Figure 106.

A, state of rest. B, state of accommodation. *a,* corneal reflexes, invariable; *b,* reflexes from the anterior surface of the lens, smaller and consequently nearer each other during accommodation (B), and nearer to those of the cornea; *c,* reflexes from the posterior surface of the lens, least luminous of all, maintaining their position and becoming but little smaller during accommodation. (*Landolt.*)

from behind. The distance between the two openings may be considered the length of a single object.

If, now, the patient looks straight ahead, maintaining clear vision of an object which is brought nearer and nearer, the image, a, from the cornea

remains unchanged. The image b becomes smaller and approaches that from the cornea. This proves that the anterior surface of the lens has moved forward and, because the image is smaller, has become more convex. The image c remains in the same position, but becomes somewhat smaller. The posterior surface of the lens, during accommodation, increases in curvature, but does not change its position.

The accommodation of the eye is produced by a change in the form of the lens. The anterior surface moves forward and its radius of curvature increases. The radius of curvature of the posterior surface increases slightly, but its position remains unchanged. The thickness of the lens is increased by .4 mm.

The table below gives the data, calculated by Gullstrand, of his exact schematic eye in a state of rest and during maximum accommodation. The powers are expressed in diopters, the linear dimensions in millimeters, and the distances are from the anterior surface of the cornea.

Position.	Accommodation Relaxed.	Maximum Accommodation.
Anterior surface of cornea	0	0
Anterior surface of lens	3.6	3.2
Posterior surface of lens	7.2	7.2
Radius of Curvature.		
Anterior surface of cornea	7.7	7.7
Anterior surface of lens	10.	5.33
Posterior surface of lens	6.	5.33

Lens System.	Accommodation Relaxed.	Maximum Accommodation.
Refracting power	19.11	33.06
Position of first principal point	5.678	5.145
Position of second principal point	5.808	5.255
Complete Optical System of the Eye.		
Refracting power	58.64	70.57
Position of first principal point	1.348	1.772
Position of second principal point	1.602	2.086
Position of first focal point	15.707	12.397
Position of second focal point	24.387	21.016

It will be seen that, where the eye is fully accommodated, the crystalline lens is biconvex with sides of equal radii of curvature.

During static refraction the anterior principal focus, Φ, lies at its farthest distance from the eye, and is nearest the eye during maximum accommodation.

The distance of the principal planes from the cornea decreases while that of the nodal points decreases. The distance between the two principal planes increases and so, of course, does that between the nodal points. But these changes are so small that they need not be considered.

During static refraction, when the eye is completely relaxed, it is adapted to its punctum remotum, P. R., which is conjugate to the retina; when it is fully accommodated it is adapted to its punctum proximum, P. P., and this and the retina are conjugate. The eye has added the *dynamic refraction* to its static refraction.

Accommodation in Emmetropia. The P. R. of an emmetropic eye is at infinity and its static refraction

$$\frac{1}{R} = \frac{1}{\infty} = 0$$

Suppose an emmetropic eye, by exerting its total accommodative power, barely distinguishes an object clearly at a distance of 10 cm. The maximum refraction is 10 D, the region or range of accommodation extends from 10 cm. to infinity and the power or amplitude of accommodation

$$a = 10 \text{ D} - 0 = 10 \text{ D}$$

Hence, we may say, for the amplitude of accommodation in emmetropia

$$a = p$$

Presbyopia. The power of accommodation gradually decreases with age. In the tenth year, according to Donders, the amplitude of accommodation is 14.50 D, which gradually decreases so that in the 30th year it has fallen to about ½. By the 40th year, when the P. P. has receded to 22 cm. ($r = 4.50$ D), the eye is said to be presbyopic. After this it becomes necessary to add a convex lens to the eye so that it may distinctly see at the usual working distance. The process goes on incessantly, passing the zero point in the 75th year.

TABLE OF THE POWER OF ACCOMMODATION
(*after Duane*)

The near point is measured from the anterior principal focus of the eye.

Age	Amplitude of Accommodation in Diopters		
	Minimum	Mean	Maximum
8	11.6	13.8	16.1
9	11.4	13.6	15.9
10	11.1	13.4	15.7
11	10.9	13.2	15.5
12	10.7	12.9	15.2
13	10.5	12.7	15
14	10.3	12.5	14.8
15	10.1	12.3	14.5
16	9.8	12	14.3
17	9.6	11.8	14.1
18	9.4	11.6	13.9
19	9.2	11.4	13.6
20	8.9	11.1	13.4
21	8.7	10.9	13.1
22	8.5	10.7	12.9
23	8.3	10.5	12.6
24	8	10.2	12.4
25	7.8	9.9	12.2
26	7.5	9.7	11.9
27	7.2	9.5	11.6
28	7	9.2	11.3
29	6.8	9	11
30	6.5	8.7	10.8
31	6.2	8.4	10.5
32	6	8.1	10.2
33	5.8	7.9	9.8
34	5.5	7.6	9.5
35	5.2	7.3	9.3
36	4.9	7	9
37	4.5	6.7	8.8
38	4.1	6.4	8.5
39	3.7	6.1	8.2
40	3.4	5.8	7.9
41	3	5.4	7.5
42	2.7	5	7.1
43	2.3	4.5	6.7
44	2.1	4	6.3
45	1.9	3.6	5.9
46	1.7	3.1	5.5
47	1.4	2.7	5
48	1.2	2.3	4.5
49	1.1	2.1	4
50	1	1.9	3.2
51	0.9	1.7	2.6
52	0.9	1.6	2.2
53	0.9	1.5	2.1
54	0.8	1.4	2
55	0.8	1.3	1.9
56	0.8	1.3	1.8
57	0.8	1.3	1.8
58	0.7	1.3	1.8
59	0.7	1.2	1.7
60	0.7	1.2	1.7
61	0.6	1.2	1.7
62	0.6	1.2	1.6
63	0.6	1.1	1.6
64 to 72	0.6	1.1 to 1	1.6

Accommodation in Myopia. Consider a myopic eye of 4 D whose P. P. equals 10 cm. In this case, the P. R. being 25 cm., the range or region of accommodation is 15 cm. The static refraction (r) being 4 D and the maximum refraction (p) 10 D, the amplitude of accommodation

$$a = 10 - 4 = 6 \text{ D}$$

Suppose, in an eye having a power of accommodation of 8 D, the P. P. is 10 cm. By the same formula we find that the degree of myopia

$$r = 10 \text{ D} - 8 \text{ D} = 2 \text{ D}$$

The image of a near object clearly seen by an axially myopic eye is larger than that seen by an accommodated emmetropic eye, because the accommodated emmetropic eye may be compared to a curvature or refractive myopia. In the case of curvature myopia the image is the same size as that seen by an accommodated emmetrope. The image is smaller when the myopic eye is corrected with a concave lens and accommodated than when uncorrected and unaccommodated.

Accommodation in Hyperopia. We have seen that the P. R. of the hyperopic eye is virtual and takes the negative sign, as does r. The formula

$$a = p - r$$

therefore, must be written

$$a = p - -r$$

or

$$a = p + r$$

for hyperopia.

The amplitude of accommodation of the hyperope must necessarily be greater than that of the myope or emmetrope. A certain amount of accommodation must be exerted to first overcome the hyperopia, that is, to make the eye emmetropic, and in addition to this there must be added the power necessary to adapt the eye to its P. P.

Suppose the P. P. of a hyperopic eye of 2 D equals 10 cm. He requires 2 D of accommodation to make his eye emmetropic, then 10 D more to render an object clear at a distance of 10 cm., or

$$a = 2\text{ D} + 10\text{ D} = 12\text{ D}$$

If the degree of hyperopia is greater than the amplitude of accommodation the eye cannot see clearly at any finite distance without the aid of a glass.

The retinal image is smaller in axial hyperopia, when accommodated, than in emmetropia. It is larger in refractive or curvature hyperopia than in axial hyperopia when both are accommodated. In any form of hyperopia the image is larger with a convex lens than when accommodated.

The Determination of the Amplitude of Accommodation. The measure of the power or amplitude of accommodation is best obtained after the eye has been made emmetropic with the proper correct-

ing lens. When this has been done the P. R. is at infinity and r = O.

Fine test type may be used as an object. Donders made use of an optometer which consisted of five fine, vertical, black wires. Duane's test object is an engraved line, ½ × 5 mm., on a white card.

The object is brought to the shortest distance from the eye at which it can, with the greatest effort, be distinctly seen. The distance is measured from a cardinal point. Donders measured from the anterior nodal point; Landolt, from the anterior principal point; and Duane, from the anterior principal focus.

The power of accommodation may be measured by having the patient look at small test type at a distance while concave lenses are placed before the eye. The strongest concave lens through which he can still clearly distinguish the type is the measure of his power of accommodation.

Scheiner's Experiment. If a small object, within or beyond the distance for which the eye is focused, is viewed through two openings, so placed in an opaque screen that their separation is less than the diameter of the pupil, the object will appear doubled. The effect is produced by the admittance of two small pencils of light which are sufficiently separated on the retina of the unaccommodated eye to be recognized. This is known as Scheiner's experiment.

The diaphragm may be made by punching two small holes, about 2 mm. apart, in a piece of cardboard or other opaque material. Look through this,

held with the holes horizontal, and focus the eye on a needle or fine wire, held vertically at the proper distance. The object will appear single and sharply defined. If, now, the object is brought closer to the eye than its P. P. there appear to be two needles. The same effect is seen when the accommodation is relaxed.

Let OO′ (Figure 107) be the openings in the screen, S, and LO and LO′ be the small pencils of

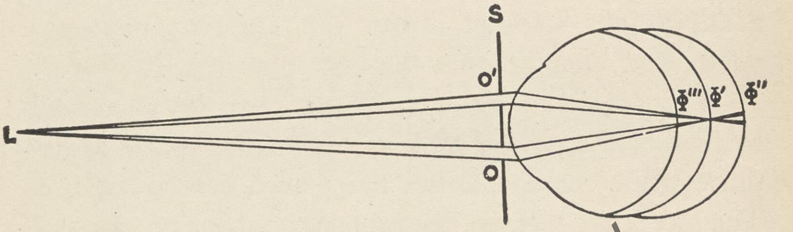


Figure 107.

light from an object-point, L. The retina is in position to receive a clear, single image when placed where the rays cross at Φ′. When the retina lies at Φ″, *i.e.,* when it lies behind the focal plane of the dioptric system, the eye is accommodated for an object closer than L and the image appears double. If, in this position, one of the openings is covered the image on the opposite side will disappear because the rays have crossed at Φ′.

If the eye is accommodated for a greater distance than L the retina will lie in the position, Φ‴, in front of the focus. If now, one of the openings is covered, the image on the same side will disappear.

CHAPTER XII.

APERTURE OF THE SYSTEM. DIFFUSION IMAGES.

A *diaphragm* or *stop* is a necessary part of any well constructed compound lens system. There may be one or more, in front of, between, or behind the dioptric apparatus, and usually so placed that the optical axis of the system passes through the center of the *stop-aperture,* perpendicular to the plane of the stop. The purpose for which stops are used is to admit, as far as possible, only such portions of a pencil of rays as are desirable to participate in the formation of the image. These rays, which are allowed to enter the system and form the useful image, are called the *effective rays.*

Every point of the object is considered the apex of a pencil of effective rays, and the base of this pencil is formed by the stop-aperture. The aperture, therefore, is the common base of all the pencils of effective rays. If the stop-aperture is circular the base is circular in shape and, of course, the pencil is a cone; but it is obvious that whatever the shape and size of the aperture, so will be the shape and size of the base of the pencil of effective rays. A large aperture allows more light to enter the instrument, making a bright image, but with less definition.

With a small aperture there will be more sharpness of detail, but at the expense of illumination.

In the eye the iris is the stop, and the pupil the stop-aperture of the system. When the pupil dilates, more light enters the eye; when it contracts there is greater definition, but the illumination is diminished. Since the iris controls the amount of light which is allowed to enter the eye it also serves as a *photostat.*

The pupil, as we see it, is really a virtual image by the cornea and anterior chamber. It appears slightly magnified and pushed forward. Consider a pupil 4 mm. in diameter, and 3.6 mm. behind a cornea whose radius of curvature is 8 mm. The anterior principal focal distance, F, of the cornea will be 24 mm. in front of its anterior surface, and the posterior principal focal distance, F′, 32 mm. behind the anterior surface. The position of the image of the pupil can be found by the formula $ll' = FF'$.

Consider L an object-point on the pupil (Figure 108) which, lying between the surface and the posterior principal focus of the cornea, Φ′, makes l negative.

$$l = 3.6 \text{ mm.} - 32 \text{ mm.} = -28.4 \text{ mm.}$$

$$FF' = 32 \text{ mm.} \times 24 \text{ mm.} = 768 \text{ mm.}$$

and

$$l' = -27 \text{ mm.}$$

The image lies 27 mm. from the anterior principal focus in a negative direction, that is, in the anterior chamber, 3 mm. from the cornea.

The size of the pupil image, if we designate the diameter of the pupil by O and the diameter of its image by I, will be

$$I = \frac{OF}{l} = \frac{32 \times 4}{28.4} = 4.5 \text{ mm.}$$

The image is called *apparent pupil.* It will be seen in Figure 108 that rays which are directed to-

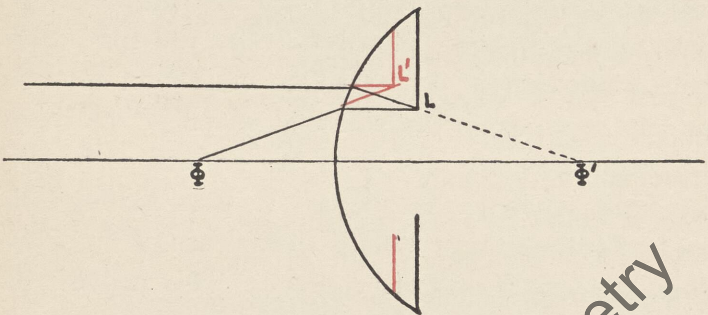


Figure 108.

ward the apparent pupil take a direction, after refraction at the cornea, towards the real pupil. The luminous pencils of effective rays, which enter the eye, are limited by this apparent pupil, called *pupil of entrance,* by Abbé.

The image of the iris and pupil by the crystalline lens would be about .1 mm. farther back than it actually is and enlarged .2 mm. This has been called the *pupil of exit,* by Abbé.

EXPERIMENT *to show the Diffusion Image of a Point.* Set up a simple optical system, consisting of

a convex lens (about 10 D), an image screen of ground-glass and a small, round, luminous object. Consider the object (for example, a 1 mm. hole in some opaque material illuminated from behind) to be a point. Carefully center and arrange these so that the object and the image-screen are conjugate.

Now pass an opaque card across the lens, either in front or behind, but close to it, gradually covering the entire surface, and at the same time notice the effect on the image. It will be observed that, while the illumination is decreased as more and more of the lens is covered, the size and shape of the image will be unchanged. Its form can be seen as long as any light whatever is allowed to pass through the lens, and the image finally disappears as a whole when the lens is entirely covered. This proves that the rays which go to form the image pass through every part of the lens.*

Note the effect on the image with a stop having an irregular, triangular, square, narrow slit or several apertures. So long as the object and screen are conjugate there will always be one image regardless of the number of apertures; its position on the screen will not change whether the stop-aperture is in the center or periphery of the stop; and its shape will be unaffected by the shape of the opening.

Almost exactly what occurs in this simple system

*As we are dealing with a limited phase of the subject, the aberrations are considered corrected and will be disregarded.

takes place in an emmetropic eye. If an eye is emmetropic or made so with the proper lens, provided there is sufficient illumination, neither the size, shape nor position of the pupil will interfere with distinct vision.

With a small pupil or stenopeic aperture there will be more sharpness of detail with less brightness; but when the aperture is very small, even though there is adequate illumination, diffraction plays a part, which, according to Tscherning, begins to be effective when the opening is less than 2 mm.

Generally speaking, large pupils do not interfere with the highest visual acuity. There may be a certain amount of glare and aberration, but this is overcome to a great extent by the structure of the retina.

Good vision does not depend on the shape of the pupil. In man, as in most animals, it is practically round, but in some of the lower animals the variability of the shape is no less remarkable than the size. This feature is sometimes made use of to distinguish certain zoological characters. The pupil may vary even in the same animal, for example the cat, where it contracts to a tiny slit in the light and dilates to a large circle in the dark. It is not uncommon to find normal vision in ragged, irregular and displaced pupils the result of disease, traumatism or operation. The same may be said of sharply defined opacities in the media when there is not too much obstruction of light.

Continuing our experiment, move the image-screen, which we consider the retina, closer or farther away so as to represent hyperopia or myopia. Instead of being clear and well defined the image is blurred or diffused. If the lens is round the *blur-image* or *diffusion-image* is round, because the rim of the lens acts as a stop-aperture. The size of the diffusion-image depends on the size of the stop-aperture and it increases as the distance of the screen or retina from the focal plane increases. In this way ametropia may be measured by the size of the diffusion-image.

Let Φ' (Figure 109) be the principal focal plane of the dioptric system and p, between O and O′ in the stop S, the exit pupil. The rays AA′ and BB′ cross in the principal focal plane to form a sharp image, and if the retina lies in front of or behind the principal focal plane, Φ''' or Φ'' for example, the image will be larger and diffused.

Designate the size of this diffusion-image by d, the distance from the pupil to Φ' by F, and the distance from Φ' to the retina of the ametropic or unaccommodated eye by l. It can be easily shown that

$$\frac{\mathrm{p}}{\mathrm{d}} = \frac{\mathrm{F}}{l}$$

and

$$\mathrm{d} = \frac{l\mathrm{p}}{\mathrm{F}}$$

If, now, we repeat what was done before with various stop-apertures we see that the diffusion-image assumes the geometric form of the aperture—erect when the image-screen lies within the focal plane, as in hyperopia, and inverted when behind. This is because the rays cross at the focal plane. If we pass a card across the lens the effect on the diffusion-image is in an opposite direction when the

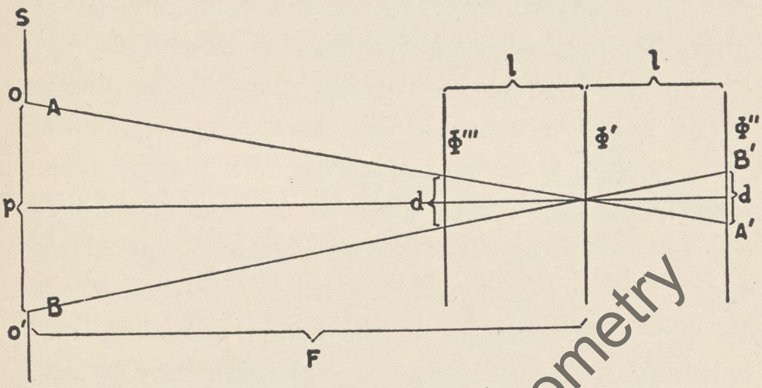


Figure 109.

distance of the image-screen is greater than the focal distance, and in the same direction when the distance is less. This same test may be done clinically to determine the form of ametropia, but it must be remembered that in the eye the movement is interpreted in the opposite direction.

Illustrate Scheiner's experiment (pages 223-224), which is performed with a stop having two apertures. There will be two diffusion-images, as in an ametropic or unaccommodated eye and if one hole is

covered, either the image on the same side or that on the opposite side will disappear according to the position of the image-screen; showing that when the image is not in focus its position *does* change with the position of the aperture. This is important, because if the image is formed anywhere except at the fovea the vision is peripheral and indistinct. Note the linear diffusion-image with a stenopeic slit aperture. With a central stop, which simulates an opacity in the media, the geometric form of the stop will be seen within the diffusion-image.

In every instance of the foregoing experiment, on accurately focusing the system, the image becomes clear, regardless of the shape or position of the stop-aperture. These observations indicate that all patients with ragged, misplaced and otherwise irregular pupils or opacities in the media should be most carefully refracted, since the confusion may be such that the vision is lowered out of all proportion to the error.

EXPERIMENT *to show the Diffusion-image of the Whole Object.* To produce the effect for the whole object use, instead of a single illuminated opening, a perforated figure. A cross formed by a number of small round holes will answer the purpose. With this object repeat all that was done before. What was seen when the object was a single opening and considered a point, now takes place for what may be considered each point of the entire object.

Unless the screen lies in the image-plane of the dioptric system a diffusion-image will be formed for each object-point (Figure 110). These diffusion-images will change in size and shape under the same conditions as in the previous experiment, and when large enough to overlap each other the whole image becomes blurred. When a card is passed across the lens each image is eclipsed as before, but the general contour of the cross remains (Figure 111). In the case where a stop-aperture is interposed each blur-

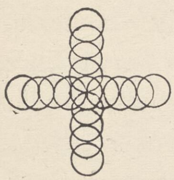


Figure 110.

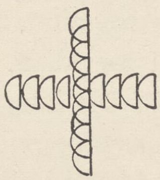


Figure 111.

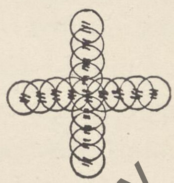


Figure 112.

image takes the geometric form of the stop-aperture. Incidently, it might be said that the term *diffusion-circle* or *blur-circle* cannot be properly used unless the pupil is circular. When a stop having two apertures is placed before or behind the lens, two diffusion-images will be formed; and, therefore, two complete crosses will be imaged on the screen. With a central stop each diffusion-image will contain a geometric figure of this stop (Figure 112).

Observe how, as before, on adding the proper correction or by placing the screen in the focal plane, each image-point of the whole image becomes clear

and distinct. This is the reason why certain opacities in the media, which are a great source of annoyance to an ametrope, may be imperceptible to the same patient wearing his proper glasses. Only when an opacity is close enough to cast a geometric shadow on the retina can its form be outlined by the emmetropic patient.

Combine a cylinder with the spherical lens and see the effect with the stenopeic slit in astigmatism. In astigmatism the image of a point is a line—a diffusion-image—and the stenopeic slit aperture, when placed in the right direction, merely shortens this line in one principal meridian. It can produce no change in the form of the image in emmetropia. A circular stenopeic aperture will answer the same purpose, even better than the slit, because it need not be so accurately placed. A circular stenopeic aperture will clear the image in any form of refractive error, provided there is sufficient light, by cutting down the size of the diffusion-image; but where this is used as a test for visual acuity in ametropia better vision can nearly always be expected with the proper lens. On the other hand, a stenopeic dot aperture may improve vision in some cases of irregular astigmatism or conical cornea while lenses do not.

CHAPTER XIII.

OPHTHALMOSCOPY. RETINOSCOPY.

The Ophthalmoscope. The ophthalmoscope is an instrument which illuminates and at the same time enables an observer to examine the interior of the eye.

The invention of the ophthalmoscope is, and should be, credited to Helmholtz in 1851; but three years previous to this an instrument, based on the principle of the modern ophthalmoscope, was made and used by Charles Babbage, who failed, however, to appreciate its usefulness.

As early as 1704, Méry observed the fundus of a cat's eye by immersing it in water, thus converting the curve of the cornea into a plane; and Brücke all but invented an ophthalmoscope when he observed the fundus through a tube placed in a flame.

Helmholtz was the first to satisfactorily explain why, under ordinary conditions, the fundus cannot be seen when it is illuminated. He showed that if the eye is exactly accommodated for any luminous point, A, the rays leaving the eye in the point A′ will travel along the same path and be united again in the point A. The object and its retinal image are in the position of conjugate foci and the rays proceeding from either point are reunited in the other. This

principle was seen in the conjugate foci of a convex lens on page 94.

Every ray in its exit from the eye follows exactly the same course as in its entrance, and the image of the retinal image is formed only at the luminous object-point. In order to see the returning rays from the illuminated fundus the observer must be

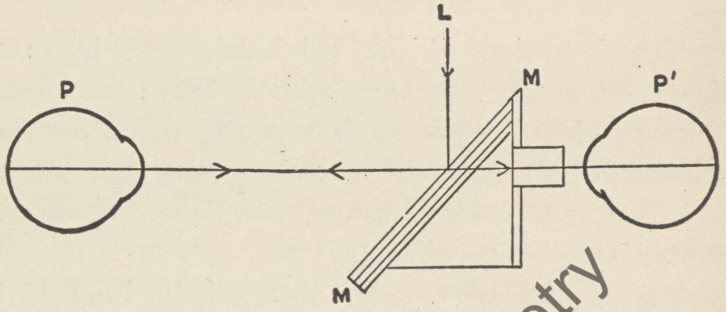


Figure 113.

placed between the illumination and the eye to be examined, without obstructing the light from the source. We are enabled to do this by means of the ophthalmoscope.

Helmholtz's ophthalmoscope consisted of three plates of glass arranged as in Figure 113. Part of the light from L passes through the plates, MM, but some is reflected into the observed eye, P, and the illuminated fundus is seen through the plates by the observer, P′.

In 1852 Reute used a concave mirror with a clear opening in the center through which the observer could see the illuminated fundus. Since Helmholtz's time ophthalmoscopes have been made in many different forms, but they all consist essentially of a mirror or prism to reflect the light into the observed eye, and a mechanical arrangement to bring various lenses before the opening. Instead of using a separate light the electric ophthalmoscope makes use of the light from a small bulb in the handle of the instrument, and for this reason is called self-luminous. In the non-luminous or reflecting ophthalmoscope a concave mirror, of a radius of curvature varying between 10 and 50 or more centimeters and having a perforation of from 2 to 4 millimeters, is used. The most popular of these, the Loring ophthalmoscope, has a concave mirror of 40 centimeters radius with a 3.5 millimeter opening.

Daylight or any good source of artificial illumination may be used. The color of the light affects the fundus picture to a certain extent and filters may be used to give desired color effects or to make artificial light resemble daylight.

Every variety of reflector has been used: reflecting prisms; metal or glass mirrors, convex plane and concave; one form of electric ophthalmoscope uses metal reflectors in a venetian blind arrangement, the observer looking through a slit; another has a tiny bulb placed in front of the opening so that the

light is thrown directly into the observed eye. The sight hole also varies in size, shape and position. The style of the mirror does not affect the refraction and the direction of the reflected rays need not be considered; they serve merely to light up the fundus. The concave mirror gives more intense illumination than a plane mirror because it concentrates the light.

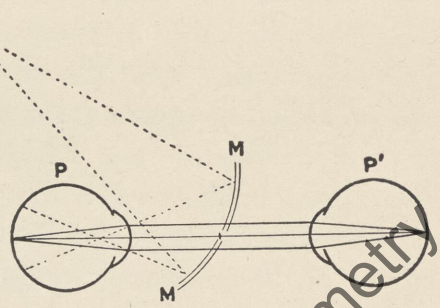


Figure 114.

On the other hand, for certain purposes, it is an advantage to have the light less concentrated, and for these a plane or even a convex mirror is used. No attempt will be made to enumerate or describe the great variety of ophthalmoscopes being used at the present time, but merely to discuss the theory which is practically the same in all.

Direct Method of Ophthalmoscopy. Rays from the illuminated fundus, that is the object, are refracted by the dioptric system of the eye and brought to a focus in some point which is conjugate to the retina. If the eye being examined is emmetropic the rays, after refraction, will be parallel, since for this eye the incident rays must be parallel in order to be focused on the retina; and if the eye of the observer is also emmetropic the emergent parallel rays will in turn be focused on his retina. Under these conditions this is all that is necessary.

The image seen by the observer (P′, Figure 114) will be erect, virtual and enlarged, as are all objects which we look at through a lens at whose focus they are situated. The rays being parallel, varying the distance between P and P′ does not affect the size of the image.

Magnification of the Image by the Direct Method. For the magnification the image formed on the retina of P′ must be compared to the same size that the object would appear to be if it were at the ordinary working distance that the observer is accustomed to use. This distance is ordinarily from 8 to 10 inches or 20 to 25 centimeters.

In Figure 115 consider a portion of the illuminated retina, LA, as the object O. All rays from any point in this object such as A will, after refraction, be parallel to each other. One ray, which is parallel to the axis within the eye, will pass through Φ, the common anterior focal point of the two emmetropic eyes. This ray will be parallel to the axis in P′. If we designate by F1 the anterior focal distance, HΦ, of P, and the anterior focal distance, H′Φ, of P′,

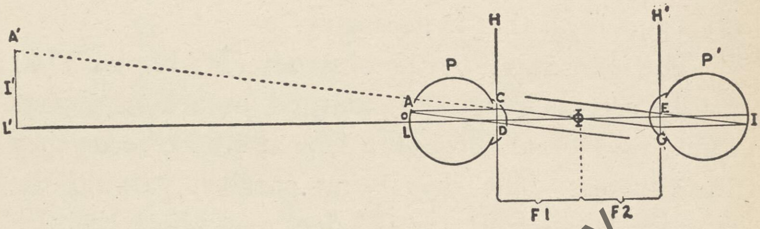


Figure 115.

by F2, the two similar triangles CDΦ and EGΦ give us the relation

$$\frac{I}{0} = \frac{F2}{F1}$$

The image is the same size as the object.

If the anterior focal distances of the eyes are equal and this distance is approximately 15 mm. the formula would be written

$$\frac{I}{0} = \frac{F2}{F1} = \frac{15}{15}$$

but if the observer views the object as though it were at a distance of 250 mm.—to L′A′—

$$\frac{I'}{I} = \frac{250}{15}$$

The apparent size of the image is 16.6 times larger than the image. The apparent magnification varies, of course, with the working distance. If either P or P′ are ametropic they must be made emmetropic with the proper correcting glass.

When the observed eye is myopic (Figure 116)

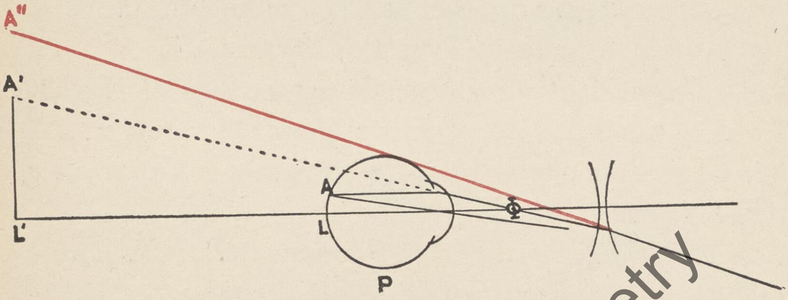


Figure 116.

the emergent rays are convergent and the image, therefore, appears larger than in emmetropia. In hyperopia, the emergent rays being divergent, the image appears smaller than that seen in emmetropia.

In Figure 116 where the required concave lens is placed before a myopic eye so that it lies in front of its Φ, which is usually the case in practice, the image appears still larger. The ray is so refracted by the lens that it appears to come from A″ and the image apparently extends from L′ to A″ instead of from L′ to A′. In a case of curvature myopia

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the magnification is greater than in axial myopia because Φ is closer to P and the rays are made more divergent by the concave lens.

In hyperopia the emergent rays are divergent and the reverse of what occurs in myopia takes place in hyperopia, both with and without a lens.

In an astigmatic eye the disc is elongated along the meridian of greatest refraction, so that, in astigmatism with the rule for instance, the greatest curvature being in the vertical meridian, the disc appears oval with its long axis vertical.

The field of view depends on the size of the observed pupil and the distance from the observer. It is larger in hyperopia than in emmetropia and smaller in myopia.

Determination of Refraction by the Direct Method. By this method of ophthalmoscopy, if the observer is emmetropic or made so by a proper correction, and both he and the observed relax their accommodations, the strongest convex or the weakest concave lens with which the details of the retina are most clearly seen is the measure of the ametropia of the observed eye. To be exact, it is necessary to make the proper allowance for the position of the correcting lens before the eye.

Astigmatism is measured by choosing a retinal vessel in each principal meridian and noting the glass necessary to render each clear, the difference between the two powers being the amount of astigma-

tism. It must be remembered that the lens which makes a vessel clear in any meridian corrects the ametropia in the meridian at right angles to it.

Indirect Method of Ophthalmoscopy. If a convex lens is held before an eye whose retina has been illuminated, the luminous rays will be focused by the lens, and a real, inverted and enlarged image will be formed in the air. This method of ophthalmoscopy was first used by Reute in 1852.

If the eye is emmetropic the emergent rays are

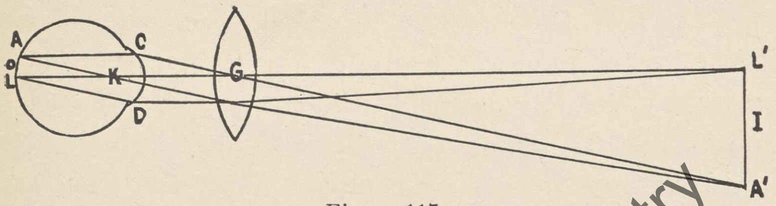

Figure 117.

parallel and the image is formed at the principal focus of the lens (Figure 117).

The similar triangles ALK and A'L'G (Figure 117) give the relation

$$\frac{I}{0} = \frac{GL'}{KL}$$

Suppose a plus 16 D lens is held before an emmetropic eye. This lens has a focal distance of 625 mm. and we know that LK is about 15 mm., so that

$$\frac{I}{0} = \frac{625}{15}$$

The magnification is about 4.16. If the lens were 13 D, a focal distance of 77 mm., the magnification would be $^{77}/_{15}$ or about 5. A weaker lens gives a greater magnification, but the field is smaller and the image is formed farther away.

While the magnification is less in the indirect than in the direct method, the field is larger. The ophthalmoscope must be held farther from P than in the direct method so that the aërial image may be looked at directly.

When the lens, G, is held in such a position before P that its posterior principal focus coincides with Φ of the eye the image of the myopic eye will be closer than the image of the emmetropic eye and that of the hyperopic eye will be farther away, but they will all be the same size. When the lens is held closer than this, as is usually the case, the image of the myopic eye is smaller and that of the hyperopic eye larger than the image of the emmetropic eye. In emmetropia the image remains the same size wherever the lens is held.

In myopia of more than 5 D a real inverted aërial image will be formed by the eye without the aid of a lens, which may be seen when the ophthalmoscope is held at the proper distance. In high degrees of hyperopia—without the aid of a lens—a large, virtual, erect image may be seen. This image is formed by projecting back the divergent rays.

Determination of the Refraction by the Indirect Method. If the interposing lens has a known focal

distance and the emergent rays from the observed eye are parallel, an inverted, real image will be formed exactly in the focal plane of the lens, at a distance F from G. If the rays are convergent as from a myopic eye, the image will be closer than F, and in a case of hyperopia where the emergent rays are divergent, the image will be at a greater distance than F of the interposing lens. In this way the P. R. of the observed eye may be calculated. The Schmidt-Rimpler optometer is based on this principal.

Retinoscopy. Retinoscopy, skiascopy or the shadow test is the most accurate objective method of measuring the ocular refraction. It is done by means of a *retinoscope* and by observation of the real or apparent movement of the retinal reflex. This method of examination has been given many names, but the term retinoscopy is that by which it is best known. It was first demonstrated by Cuignet in 1873, and developed by Parent, who first explained it about eight years later.

A retinoscope is merely a plane or concave mirror with a central perforation through which the observer views the apparent movement of the reflex on the various rotations of the mirror. In retinoscopy, as in ophthalmoscopy, since the illuminated fundus is the object, we have only to find the point where these rays are brought to a focus in order to know where the punctum remotum of the eye under observation is located.

It makes no difference whether the mirror is plane or concave; it need only be remembered that

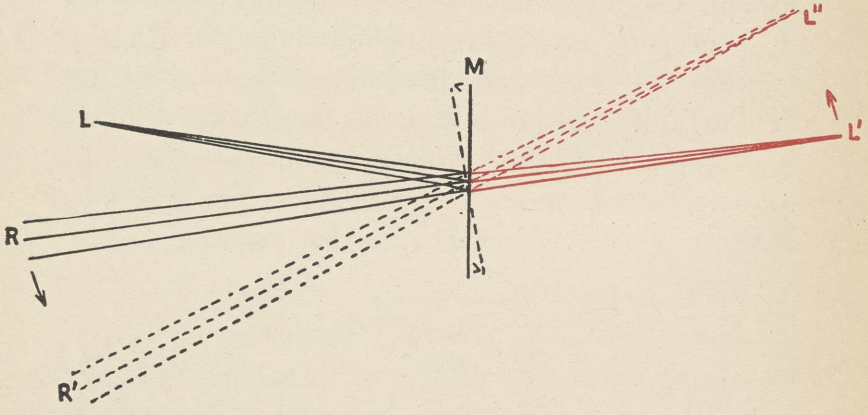

Figure 118.

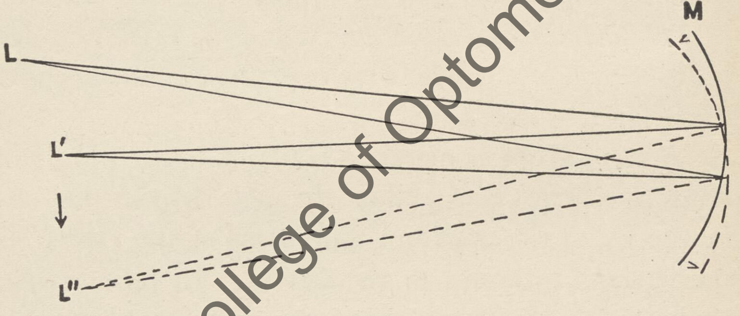

Figure 119.

when a plane mirror is tilted the direction of the reflected pencil (MR, Figure 118), which is divergent, is thrown in the same direction. The virtual image

of the illuminant, L, by the mirror, M, is directed from L′ to L″. With the concave mirror (Figure 119) the rays from L are focused in a point, L′, between the observed and the observer. This is a real image and, on tilting the mirror in the direction indicated by the dotted line, moves from L′ to L″. The rays having crossed at the focus will be moved across the fundus in an opposite direction to that in which the mirror is rotated.

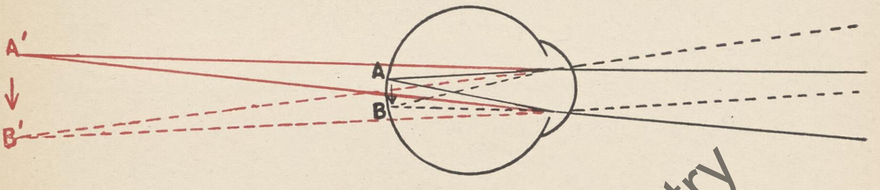

Figure 120.

Figure 120 represents a hyperopic eye. Rays from a point A will, after refraction, be divergent and a virtual, erect image will be formed at A′ from which the emergent rays appear to diverge. If the luminous object-point on the retina, A, be moved to B by rotating the mirror (in the same direction if plane and in an opposite direction if concave), the image being virtual will move from A′ to B′, the same direction that the object moves.

In myopia (Figure 121) the rays from A, on the retina, will be focused in A′, a real inverted image, and if the object is moved to B, the image, B′, will be moved in an opposite direction to that taken by the object.

A and A′ are conjugate foci and we know that if either is moved the other must move according to the laws for conjugate foci. In this case A′ is the punctum remotum of the eye and if the observer is

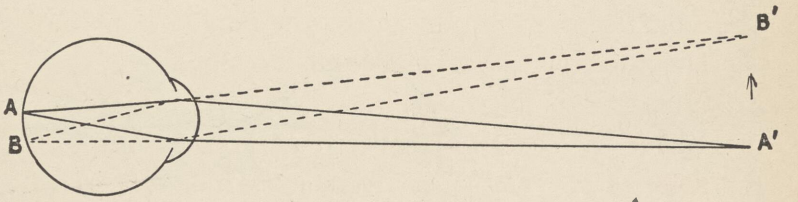

Figure 121.

placed here, there will be no movement of the fundus reflex. This point is called the *point of reversal.* If the observer is closer to the eye than A′ the conjugate point will be behind the retina as in hyperopia and will move in the same direction. If the observer is farther than A′ the reflex will move against as in myopia.

It can be seen from the foregoing that the observed eye must be overcorrected so that its punctum remotum will coincide with the distance of the observer. If this distance is one meter and the observed eye has a myopia of one diopter, there will be no movement because the punctum remotum is

situated one meter from this eye. If the eye is emmetropic it will be necessary to make an artificial myopia of one diopter by adding a plus one diopter lens. If the operator sits at a distance of two meters it is necessary to add a plus one-half diopter lens; if the distance is one-half meter the added lens will be plus two diopters. This overcorrection must be deducted from the final findings.

The retina of the observed eye is not seen. The examiner's eye is focused in the pupillary plane and the reflex is seen to move across the pupillary space.

The reflex is dull, small and its movement slow in high errors. The brightness, size and rapidity of movement increases as the error decreases until, at the point of reversal, its movement is infinitely great and the reflex fills the whole pupillary space. It is this which accounts for the band of light in the direction of the least error, or along the axis, in astigmatism.

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